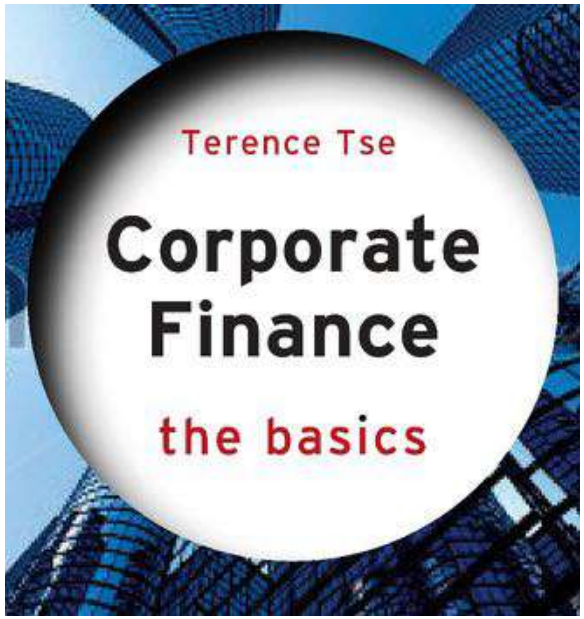


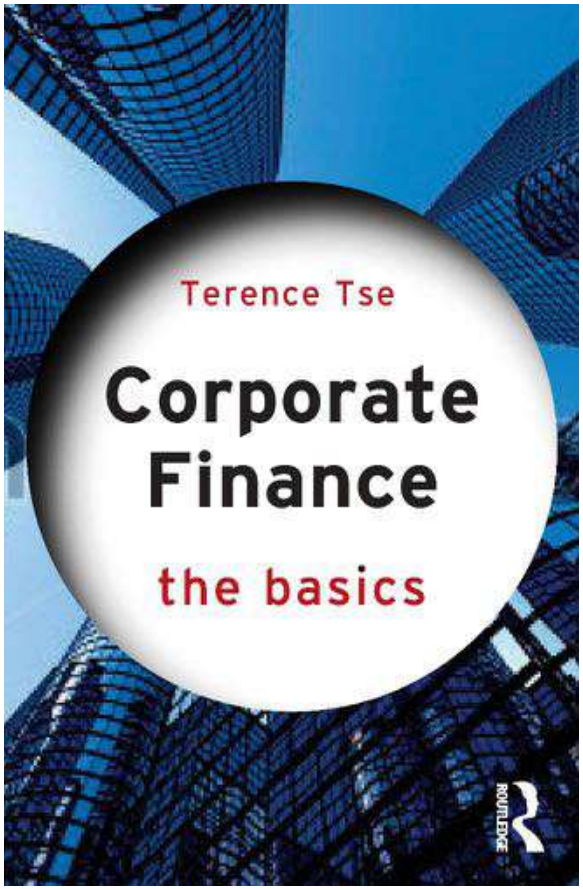
Corporate Finance

Book 2

Dr Terence Tse

www.linkedin.com/in/terencetse

NPV positive? Now available for sale



Corporate Finance: The Basics *is a concise introduction to the inner workings of finance at the company level. It aims to take the fear out of corporate finance and add the fun in, presenting the subject in a way that is simple to grasp and easy to digest. Its aim is to explain – and demystify – the essential ideas of corporate finance, avoiding the heavy use of maths and formulae. The calculations and figures in the book are purely to illustrate fundamental concepts, appealing to readers' common sense, rather than stretch their ability to do "number-crunching".*

Topic 1: Risk and Return

Defining returns

Risk and return always go together. We will therefore direct our attention to risk. But first it is necessary to provide a few comments on return, which represents the gain or loss on an investment

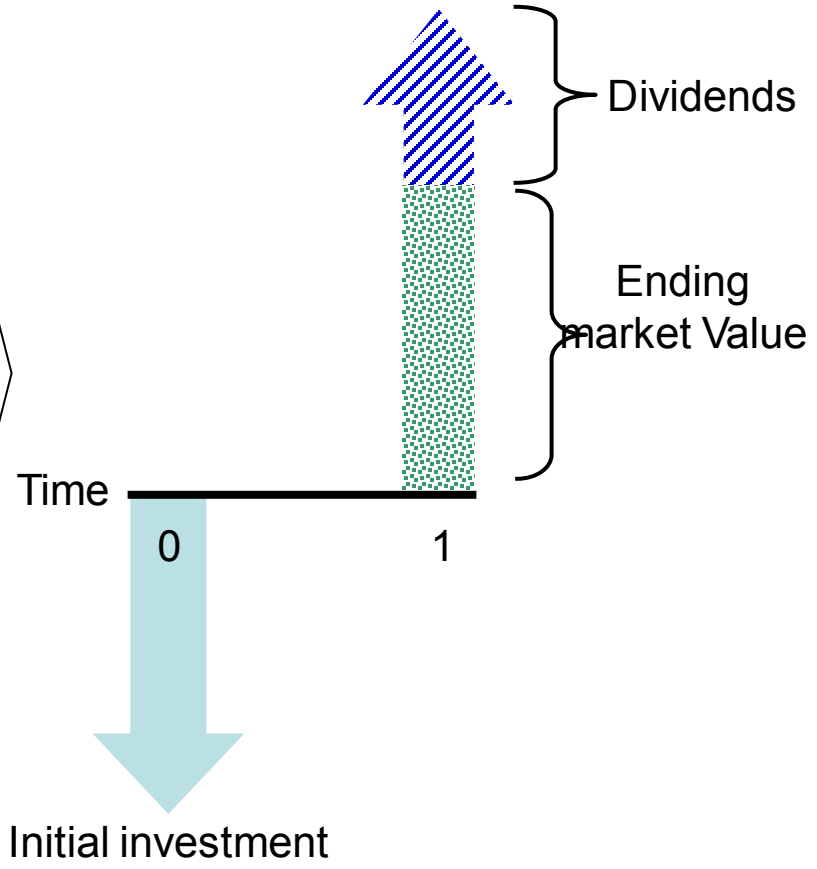
- Rate of return can be easily calculated by:

$$\frac{(W - I)}{I}$$

Whereas W is the wealth at the end of a period and I is the initial wealth

There are at least 2 ways to present return on an investment, which include 1) dollar (or pound) returns …

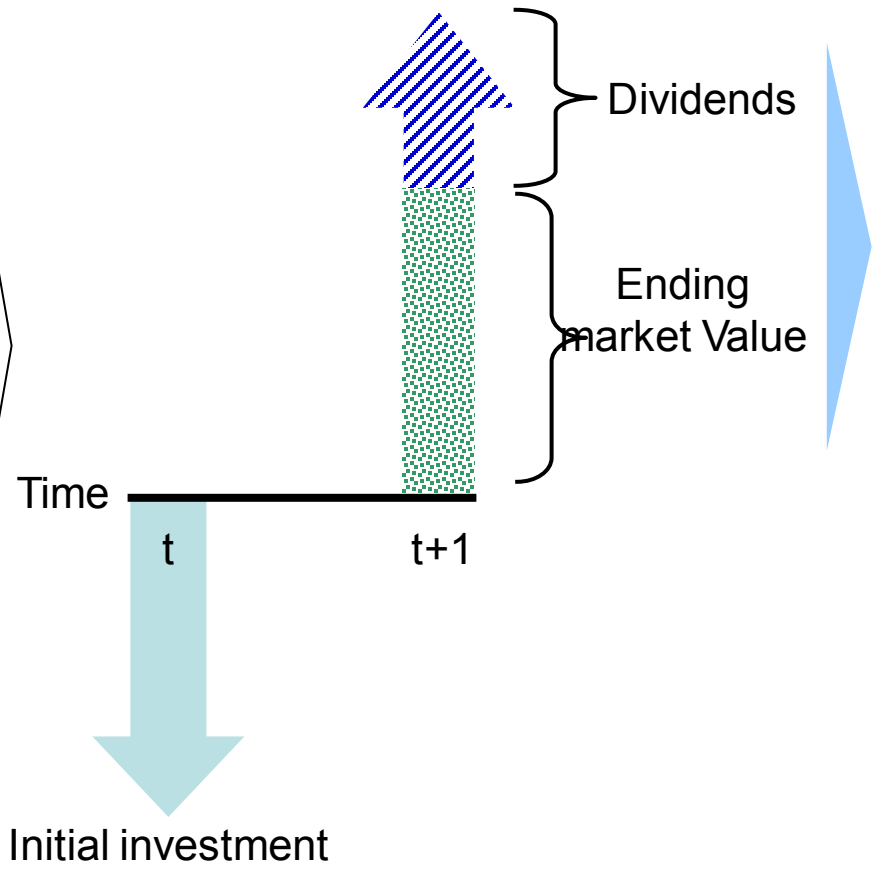
- Dollar returns – returns in terms of the absolute amount of money – on the investment in a share is the *sum* of the dividend income *and* the capital gain or loss on the investment



- Dollar returns is therefore the money received from owning the share *plus* the change in the market value of the share

… and 2) percentage returns

- Percentage returns, on the other hand, provide a more convenient way of summarising the information on returns because percentages can be applied to any amount invested
- So, it is about asking how much return we can get for each pound invested



- Percentage return on the investment in a share is the total of *dividend yield* and *capital gain (or loss) expressed in percentage change*
- The percentage return on an investment is therefore calculated with the following formula:

$$\frac{Div_{t+1}}{P_t} + \frac{(P_{t+1} - P_t)}{P_t}$$

Example 1

- Examine the return of a share in Vivendi Universal
- What can you say about the return on investment from owning this company's share?

Source : Yahoo! Finance

Example 2

Aspects	Descriptions
Problem	• In October 2006, you bought 100 Vivendi shares for €30 each. Due to various reasons, you had to sell them in October 2008 and managed to fetch only €19 a share. You did not receive any dividend. What is the return of this investment?
Solution	• The initial investment was €3,000. At the time you sold them, they were only worth €1,900 • Therefore the loss incurred on you is: €1,900 - €3,000 = (€1,100) • The rate of return in percentage is therefore: $\frac{(1,100)}{3,000} = \frac{1,900}{3,000} - 1 = (36.67\%)$

Topic 1: Risk and Return

Measuring risk and return

In addition to return, an investor needs to consider the risk when assessing an investment opportunity

- Risk is defined as the chance of the final result being different from the expected result

- In the context of business and finance, the investors are more concerned with the loss or downside risk
- Assets, both real and financial, which have a greater chance of actual returns being different from the expected returns, are considered more risky

Please note that risk can be 'good if the result turns out to be better/greater than the expected value

The standard measures for risk are variance and standard deviation

- To evaluate risk, one has to quantify it
- However, there is no perfect way of measuring risk
- In finance, the most commonly-used methods include:

Concepts

Variance (σ^2 or VAR)

- The average of squared deviations around the mean:

$$\sigma^2 \text{ or } VAR = \frac{(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + \ldots + (R_T - \overline{R})^2}{N-1}$$

where R_1 is the actual return and $\overline{R}$ is the expected return and N is the number of observations

Usually you use N-1 when there are observations from a sample (and not actual population, in which case N is used)

Standard deviation (σ)

- Square root of variance
- Volatility is measured by standard deviation of annual returns

$$\sigma = \sqrt{VAR}$$

Deviations are squared and then square-rooted in order to prevent them from cross cancelling as a result of having both positive and negative numbers

Descriptions

Example

Aspects	Descriptions
Problem	• What is the mean, variance and standard deviation of Microsoft's share over this period?
Solution	

	Annualised returns
1 Sep 03	36.8%
1 Mar 03	23.4%
1 Sep 02	-47.4%
1 Mar 02	39.0%
1 Sep 01	-12.6%
1 Mar 01	-17.7%
1 Sep 00	-7.1%
1 Mar 00	-52.8%
1 Sep 99	3.3%
1 Mar 99	164.7%

Mean	13.0%
Variance	0.38

Exercise

Aspects	Descriptions
Problem	• What is the expected return, variance and standard deviation of shares X and Y below?

Probability	Return	
	Share X	Share Y
0.2	11%	-3%
0.2	9%	15%
0.2	25%	2%
0.2	7%	20%
0.2	-2%	6%

The risk of a financial asset can therefore be measured by how far the actual returns deviate from the expected return

- A financial asset or security can be defined by its risk-return characteristics

- Return can be measured by the expected return, which is calculated by the weighted average of possible returns
- Risk, on the other hand, can be measured by the variability of its returns in relation to its expected return
- As we shall see, historical risk is measured by the standard deviation of the real returns in relation to its average return

- Hence, the higher the standard deviation (and variance), the higher the risk of the asset

While we can measure the risk of an asset, we should also consider the risk appetite of individual investors

- Different people have different inclinations to take risk
- It is possible to place them into 3 broad categories

Aspects	Descriptions
Problem	• Suppose you have the choice between receiving $9.5 million with certainty or an asset which pays $20 million with a probability of 50%. Which of these two alternatives would you choose? Would your answer be different if the certain amount is $11 million?
Solution	• If you prefer $9.5 million, you are risk-adverse • If you are indifferent between $10 million and the risky return, you are risk neutral • If you prefer the risky payment of $11 million, you are a risk lover

Note that normally investors prefer a certain payment to risky one with the same mean

The 3 categories can be depicted by the utility functions of the different level of risk aversion

- The expected utility of wealth is an important criterion to take into account
- The utility of an additional € is less important when you have already one million compared if you have nothing
- In this case, we define a concave utility function
- The opposite is true for a convex utility function

Concepts

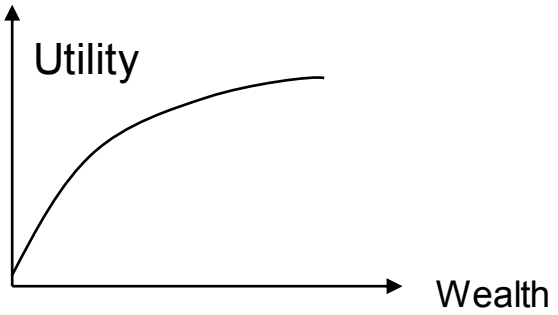
Risk-adverse

Risk-neutral

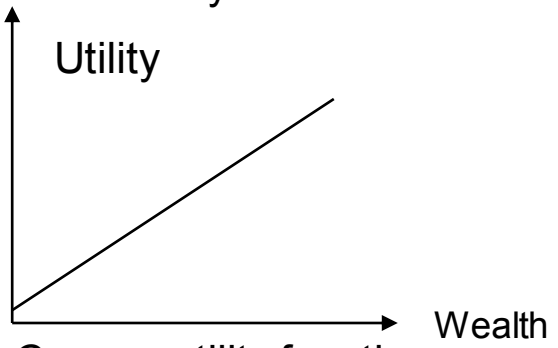
Risk-loving

Descriptions

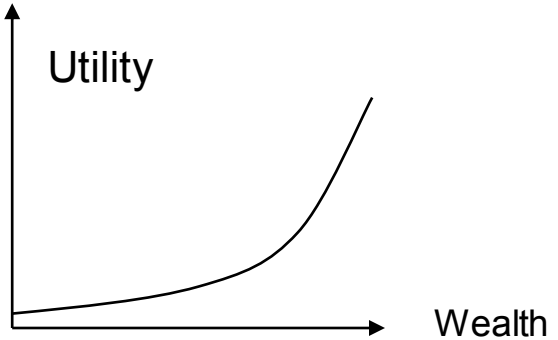
- Concave utility function



- Linear utility function



- Convex utility function

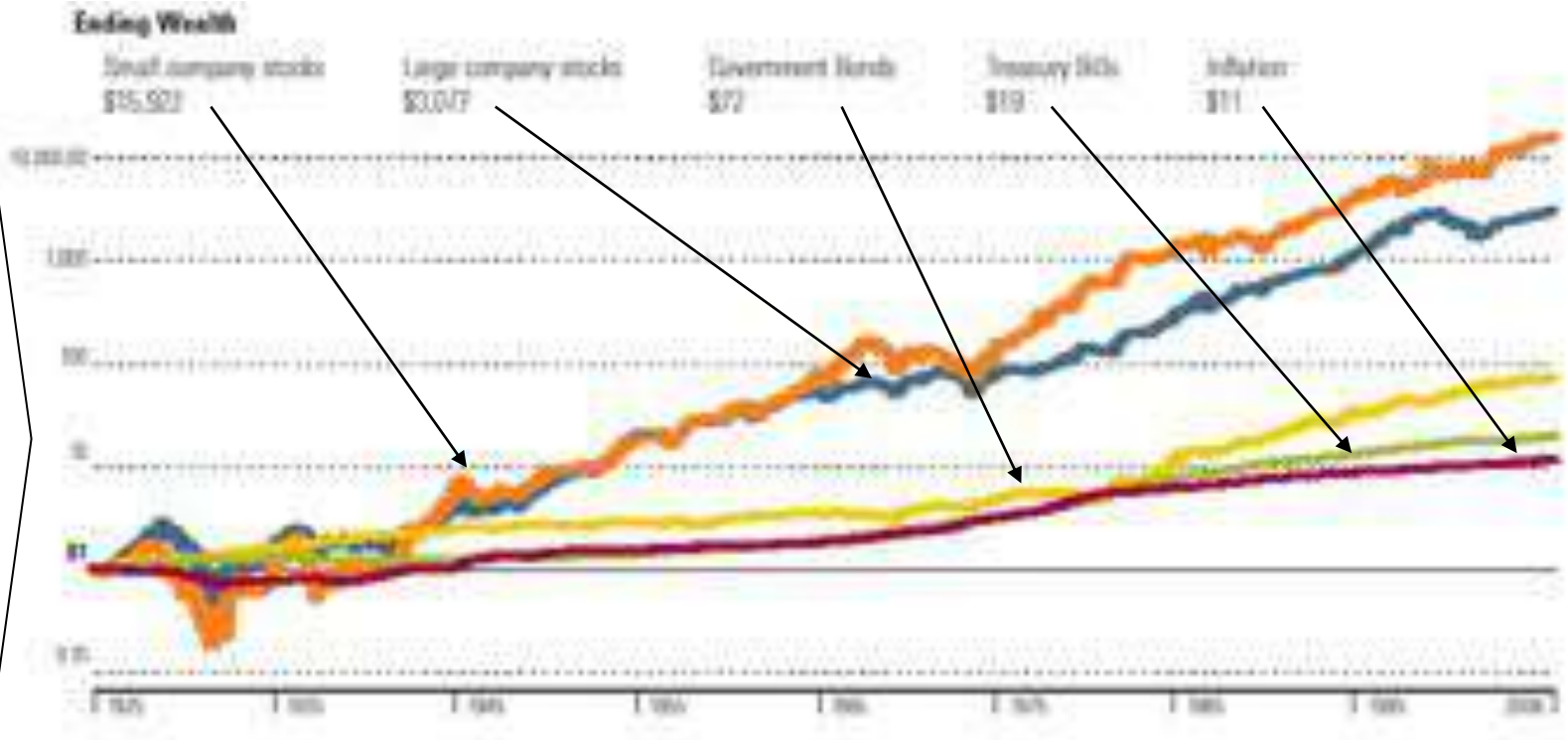


Topic 1: Risk and Return

Risk and risk premiums in financial markets

Since risk and return go hand-in-hand, it can be expected that a higher return is necessary to compensate a higher level of risk. Therefore, while different types of investment offer different returns …

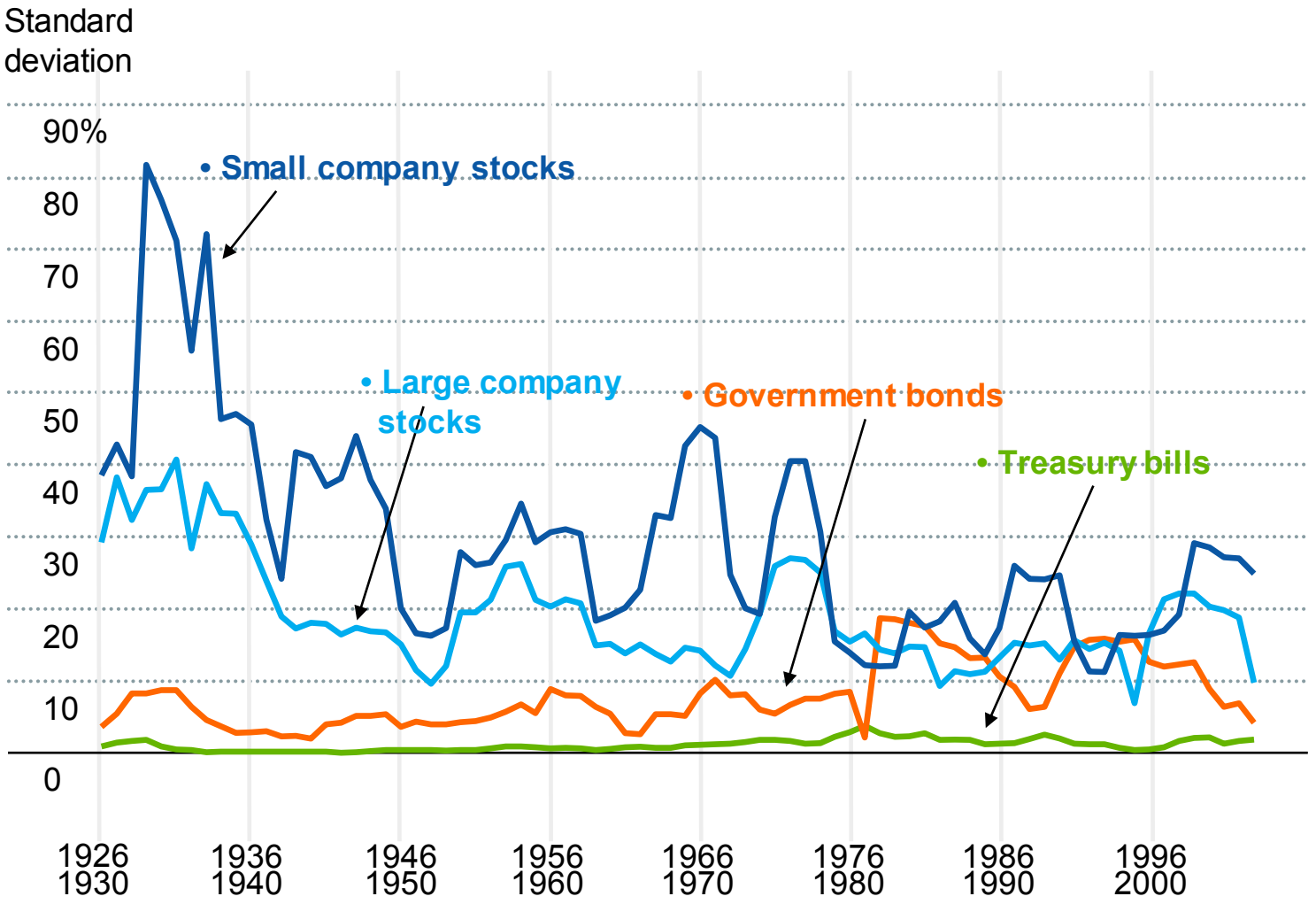
- If you invested $1 in 1925, your end-wealth will be very different if the $1 is invested in small company stocks, large company stocks, T-bonds and T-bills



Source : Ibbotson Associates (2006) Stocks, Bonds, Bills, and Inflation 2006 Yearbook

…they carry different levels of risk on returns

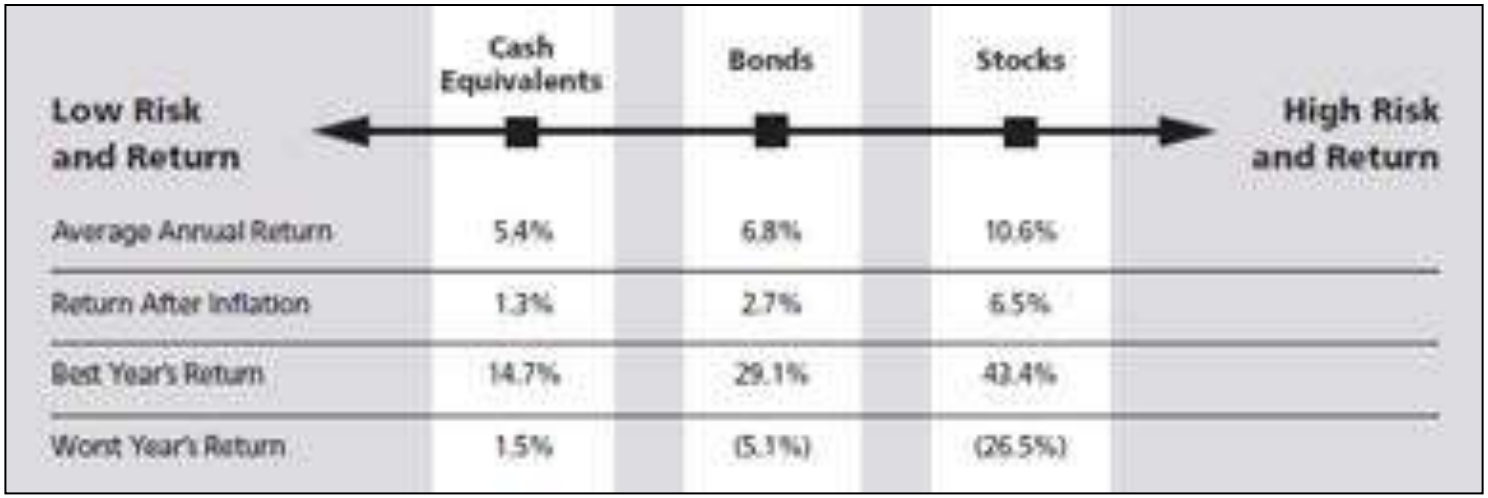
- While the common stocks offer a much higher returns than T-bonds and T-bills, the risk of the return associated with common stock is also higher



Source : Ibbotson Associates (2006) Stocks, Bonds, Bills, and Inflation 2006 Yearbook

In other words, investors will need to assume more risk if they wish to obtain a higher return

- Assets such as shares with a higher potential gain or a higher potential loss
- In comparison, cash equivalents have a lower potential gain but also a lower downside risk

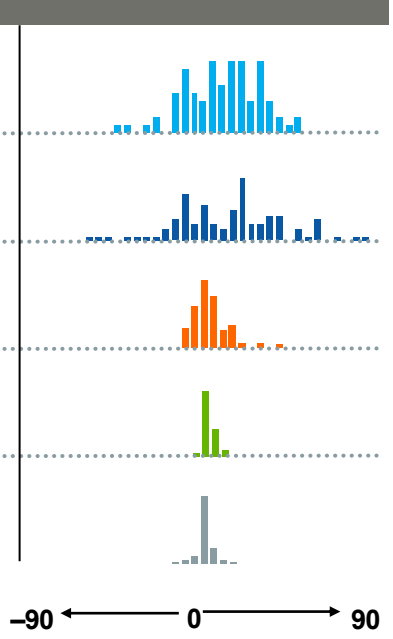


	Cash Equivalents	Bonds	Stocks
Average Annual Return	5.4%	6.8%	10.6%
Return After Inflation	1.3%	2.7%	6.5%
Best Year's Return	14.7%	29.1%	43.4%
Worst Year's Return	1.5%	(5.1%)	(26.5%)

Source : Ibbotson Associates (2007) Stocks, Bonds, Bills, and Inflation 2006 Yearbook

In a nutshell, return is proportional to the risk carried by the asset

	Arithmetic annual return	Risk (standard deviation)
Large firm stocks	12.3%	20.0%
Small firm stocks	17.1%	32.6%
Government bonds	5.8%	9.2%
Treasury bills	3.8%	3.1%
Inflation	3.1%	4.2%

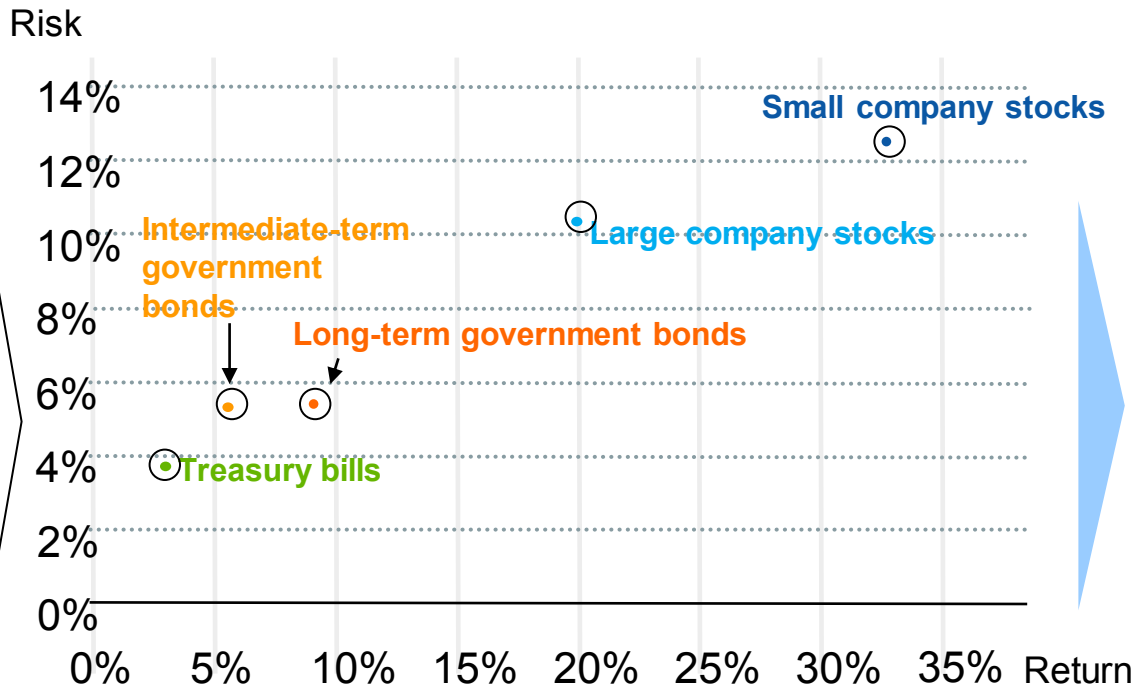


Source : Ibbotson Associates (2006) Stocks, Bonds, Bills, and Inflation 2006 Yearbook

- Small company shares provide on average better return than any other asset classes
- But they also have the highest risk (standard deviation)
- One can also observe that the shares of these companies have a wider distribution (spread) of returns (in other words, greater variability and volatility)
- T-bills, on the other hand, are far less risky buy also yield far less return

Hence, when evaluating an investment, it is necessary to consider the trade-off between risk and return. This means if you invest in a risky asset, you should expect a higher return to compensate for the additional risk you are bearing

- Once again, an investment in company stocks is more risky than an investment in T-bills



- Therefore, by taking on the additional risk, you should be paid a *risk premium*
- The risk premium is the difference between the expected return on the shares and the return on the T-bills

Source : Ibbotson Associates (2006) Stocks, Bonds, Bills, and Inflation 2006 Yearbook

Example

Aspects	Description
Problem	• If the return on treasury bills is 4%, what is the risk premium on the share of a small company?
Solution	• We can earn, according to the previous graph, 12.8% on a company share • Therefore, the difference between the this rate and the return on T-bills – 8.8% – is the risk premium

Empirical evidence from prior research on the financial markets has shown strong supports to the concepts described above

- It has been found in previous research that:

- In an efficient market, a rational investor would require a higher return on a riskier investment
- Required return is equal to the combination of risk-free rate and risk premium (to be discussed in the next sessions)
- If security A is more risky than security B, the return on A should be higher than the return on B

Topic 1: Risk and Return

Distribution of return on stocks

While we can calculate the average expected return and risk, we would often like to know how returns are distributed

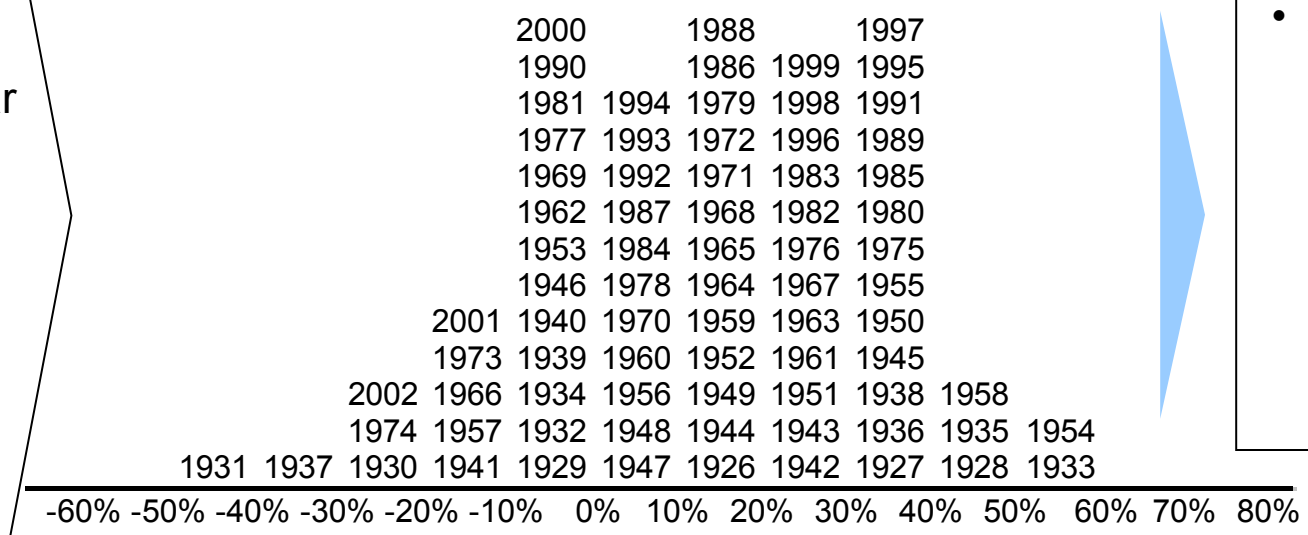
- The mean and variance of returns for different investments give some indications of risk
- However, often we would like to have more information such as:

- *What is the probability to have a return below x%?*
- *What is the maximum that we can expect to get 95% of the time?*

- To answer these questions, we have to know the exact distribution of returns
- But in practice, we often have to suppose that the return distributions are normal (also known as Gaussian)
- The normal distribution can be characterised by the mean and the variance

Historical share performance shows that returns tend to distribute around an average

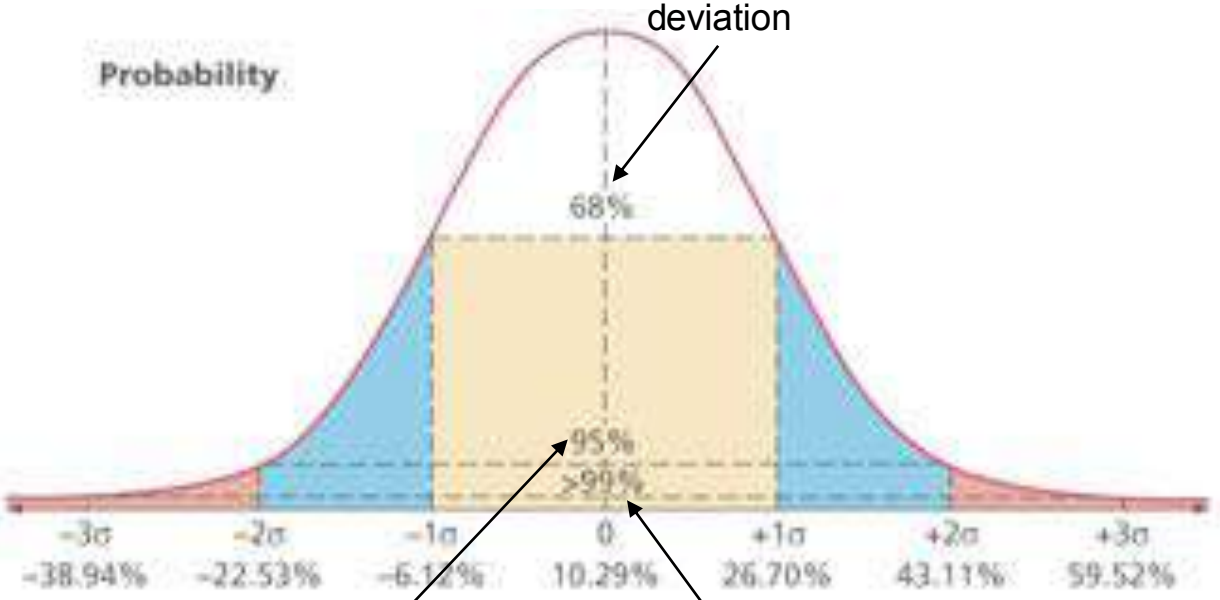
- Plotting the return of large US companies of each year from 1926 to 2002, it can be observed that returns tend to be distributed around an average



- A similar frequency distribution is also observed in small company shares

If the sample is sufficiently sizable, returns will be normally distributed

- If we were to keep on generating observations for a long time period, the aberrations in the sample would disappear, and the actual historical distribution would start to look like the underlying theoretical (normal or Gaussian) distribution
- Normal distribution looks like a bell-shaped curve



The probability to be between the mean plus or minus 1 standard deviation

The probability to be between the mean plus or minus 2 standard deviations

The probability to be between the mean plus or minus 3 standard deviations

Topic 1: Risk and Return

Calculating a 2-asset portfolio

So far, we have looked at the risk and return on a single financial asset. What would be the risk and return of a portfolio with 2 or more assets in it?

- An investment portfolio refers to any combination of financial assets
- Mutual funds, retirement funds, hedge funds and asset managements are examples of portfolios

- Institutional investors invest in a portfolio of financial assets
- They prefer to diversify (i.e. do not want to put all your eggs in the same basket)
- Otherwise they could suffer from the consequence of poor performance of the asset in question – it is simply better to “spread your bet”
- Financial markets are dominated by institutional investors

To calculate the return on a portfolio, we take the expected return of each asset according to the proportion they have within in a portfolio

- Return of a portfolio with 2 assets can be calculated as such:

The expected rate of return on a portfolio= a x E(X) + b x E(Y)

- Portion of asset X in the portfolio
- Expected return of X
- Portion of asset Y in the portfolio
- Expected return of X

- The return on a portfolio is therefore simply the weighted expected return of individual assets

On the other hand, to calculate the risk associated with a portfolio, in addition to variances and standard deviation of individual assets, it is necessary to consider how these assets will affect each other

- To consider such effect, we need to introduce 1) covariance and 2) correlation to measure the relationship between the return on one asset and the return on another
- Both covariance and correlation measure how two random variables are related
- First, covariance:

1) Covariance

- If you say that 2 items tend to vary together, then you are talking about the covariance between the 2 items which can be a positive or negative covariance
- Therefore, if two variables tend to move in the same direction, then the covariance between the two should be positive

$$\mathrm{Cov}(X, Y) \text{ or } \sigma_{XY}$$

$$= \sum_{n=1}^{N} P_n (X_n - \bar{X})(Y_n - \bar{Y})$$

where

P_n is the weight or probability

$\bar{X}$ is the mean of X

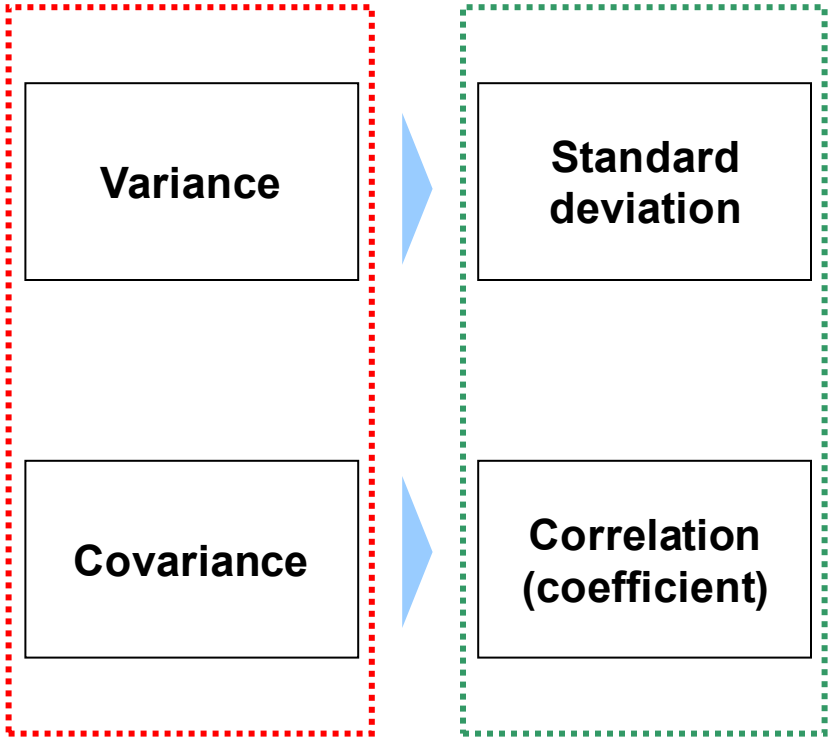
$\bar{Y}$ is the mean of Y

Covariance and correlation must also be taken into consideration when calculating the risk associated with a portfolio

- One can view covariance as the following

Measurement of dispersion

Measurement of relationship



Difficult to understand

Easier to understand (hence why we mostly use this for calculations)

Covariance and correlation must also be taken into consideration when calculating the risk associated with a portfolio (cont'd)

- Since covariance cannot be easily understood intuitively, so we translate it into something that we can understand

2) Correlation

- When you say that 2 items correlate, you are saying that the change in one item affects a change in another
- You will always talk about correlation as a range between -1 and 1

Correlation coefficient or ρ_{XY}

$$= \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

If we re-arrange it,

$$\rho_{XY}\sigma_X\sigma_Y = \sigma_{XY}$$

Covariance and correlation must also be taken into consideration when calculating the risk associated with a portfolio(cont'd)

- Let us go through a simple example to hopefully clarify all of the above
- Notice in this example that we can expect correlation to be 1.0 because Y is always greater than X by 2

	X	Y	$(X-X_{mean})(Y-Y_{mean})$
	7	9	
	3	5	
	5	7	
Mean			
SD			
Correlation			

Taking all of the above considerations into account, it is possible to calculate the return and the risk of a 2-asset portfolio

Example

- In a 2-asset portfolio called *p*, there is *a* portion of asset *X* and *b* portion of asset *Y*
- The return and risk of this portfolio are:

Return

Expected return on the portfolio

$$E(R_P) = aE(X) + bE(Y)$$

Risk

Variance of the portfolio

$$Var(R_P) \text{ or } \sigma_P^2 = Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$$

Correlation coefficient

$$\rho_{XY} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y} \text{ (also if re-arranged } \rho_{XY}\sigma_X\sigma_Y)$$

Combining the correlation coefficient, the portfolio variance can be written as:

$$\sigma_P^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho_{XY}\sigma_X\sigma_Y$$

Standard deviation of a portfolio is therefore: $\sqrt{\sigma_p^2}$

We can break down the portfolio variance formula presented above to show that the variance of a 2-asset portfolio is in fact the sum the following 4 boxes

	Asset A	Asset B
Asset A	$a^2\sigma_X^2$	$ab\sigma_{XY} = ab\rho_{XY}\sigma_X\sigma_Y$
Asset B	$ab\sigma_{XY} = ab\rho_{XY}\sigma_X\sigma_Y$	$b^2\sigma_Y^2$

- By taking the square root of the portfolio variance calculated here, you can find out the standard deviation or the risk of the portfolio

Adding them all up:

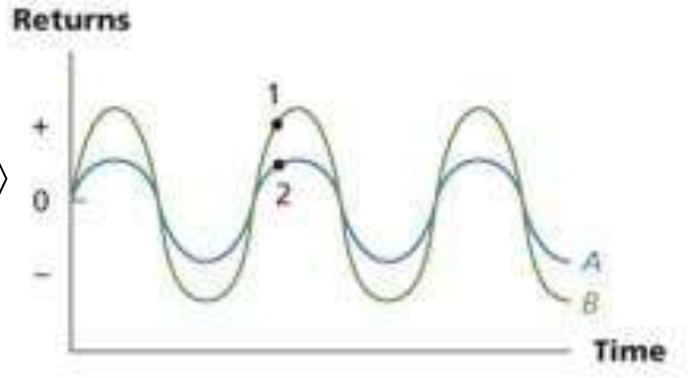
$$\sigma_P^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho_{XY}\sigma_X\sigma_Y$$

Exercise

Aspects	Description
Problem 1	• Suppose that 60% of your portfolio is invested in ExxonMobil and the remainder is invested in Coca Cola. You expect that over the coming year, ExxonMobil will give a return of 10% and Coke 15% 1. What is the expected return? 2. What is the standard deviation of the portfolio if the $\sigma_{ExxonMobil}$ is 17.2% and the $\sigma_{Coca\text{-}Cola}$ is 27.3% if the coefficient of correlation is 1?
Problem 2	• Suppose you invest £55 in Bristol-Myers and £45 in McDonald's with the return of Bristol-Myers is 10% and McDonald's 20% 1. What is the expected return? 2. What is the standard deviation of the portfolio if the $\sigma_{Bristol\text{-}Myers}$ is 17.1% and the $\sigma_{McDonald's}$ is 20.8% if the correlation coefficient is 1? 3. What happens if the correlation coefficient is 0 4. What happens if the correlation coefficient is -1?

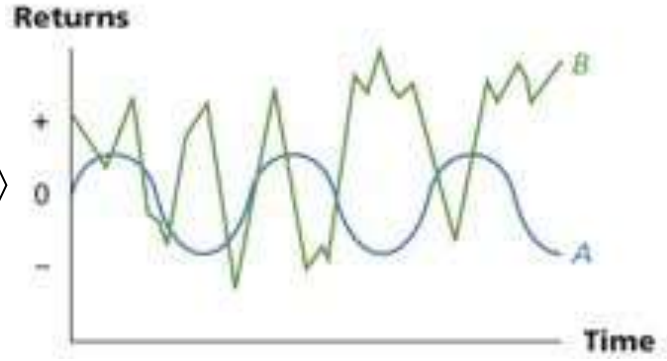
Pictorially, we can see how the different correlation coefficients work

- Perfect positive correlation (correlation = 1)



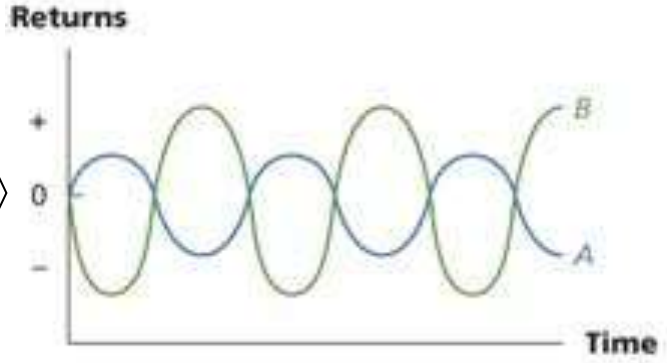
- Both the return on security A and the return on security B are higher than average at the same time and lower than average at the same time
- In other words, when the return on security A is positive, the return on security B also tends to be positive

- No correlation (correlation = 0)



- The return on security A is completely unrelated to the return on security B
- In other words, the fact that the return on security A is positive or negative does not say anything how likely it is that the return on security B will be positive or negative

- Perfect negative correlation (correlation = -1)



- Security A has a higher than average return when security B has a lower than average return, and vice versa
- In other words, when the return on security A is positive, the return on security B will always be negative

The results from the previous exercises highlight the important effects of diversification

- When the correlation coefficient changes from 1 to 0 to -1, it can be seen that portfolio standard deviation decreases, eventually to 0

- This suggests that by combining two assets that are not perfectly correlated, it is possible to achieve the *effects of diversification* which means:
  - While the portfolio returns are a weighted average of the expected returns on the individual assets, the portfolio standard deviation is less than the weighted average risk of the individual investments

- In other words, the decrease in returns as a result of including a less risky asset in a portfolio is much less than the decrease in risk
- When assets are held in a portfolio, the risk of the portfolio is lower because a portion of the risk is diversified away
- If we assume that investors are risk adverse, they would prefer to invest in portfolios rather than in single assets
- The effect of diversification depends upon the extent to which the returns on assets move together
- This movement is measured by the correlation between the returns on the assets

Diversification therefore allows investors to lower more risk while losing less on the expected return

Example

- Share X has a higher return but also higher risk, whereas Share Y has a lower return but also a lower risk

Weight of share X	Weight of share Y	Expected portfolio return	Portfolio standard deviation
100%	0%	10.0%	8.72%
75%	25%	9.5%	6.18%
50%	50%	9.0%	4.97%
25%	75%	8.5%	5.96%
0%	100%	8.0%	8.41%

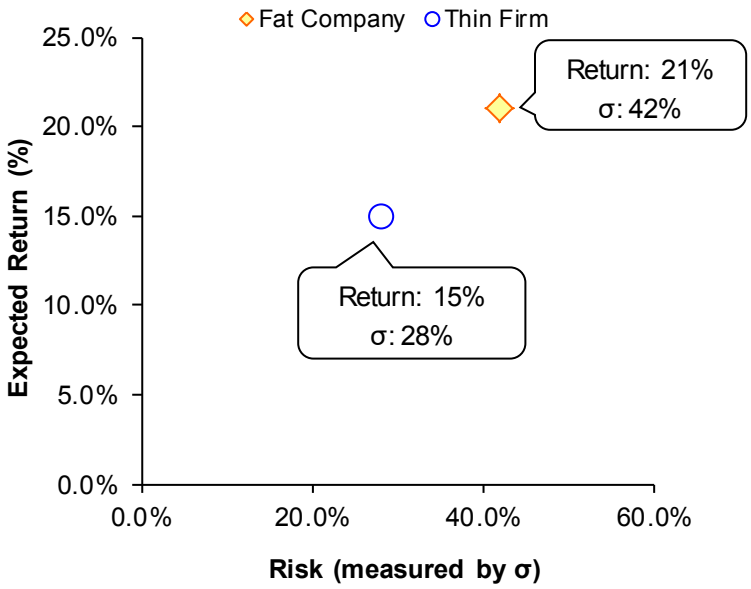
- The 50/50 portfolio has a lower expected return than that of share X, yet the portfolio has a much lower risk (in the form of standard deviation)
- Indeed, the portfolio has the lowest risk among all combinations
- Diversification forms the basis of portfolio theory

Topic 2: Portfolio Theory

Portfolio with two securities

We learned from our previous session that there is a relationship between risk and return

- Example
- Here are 2 shares, Fat Company and Thin Firm
- They have different risk and return characteristics



- It can be easily noted that Fat Company, which has a higher expected return, carries greater risk (i.e. higher probability of deviating from the expected return)
- On the other hand, Thin Firm's lower risk is compensated by a lower return

By putting together 2 assets that do not have a high correlation, an investor can reduce the risk of a portfolio

Example

- If we put the 2 securities together in a portfolio (let's call it portfolio 1)
- Let us assume that the correlation between the 2 shares is 0.4

PORTFOLIO 1

Shares	% of portfolio	Average return	σ
Thin Firm	60.0%	15.0%	28.0%
Fat Company	40.0%	21.0%	42.0%
Coefficient correlation		0.4	

- Doing the calculations, the expected return of the portfolio is 17.4%
- The standard deviation of the portfolio, on the other hand, is 28.1%
- Note that the risk of portfolio 1 is just a little bit higher than Thin Firm by itself, but the expected return on the portfolio is far higher than Thin Firm by itself
- In this example, we have started to see how by combining securities, we can get better return or lower risk

By adding a third asset into the portfolio, it is possible to further diversify the risk

- Now we add another asset called Slim Corp into portfolio 1
- Let us assume the correlation between the 3rd security and portfolio 1 is 0.3

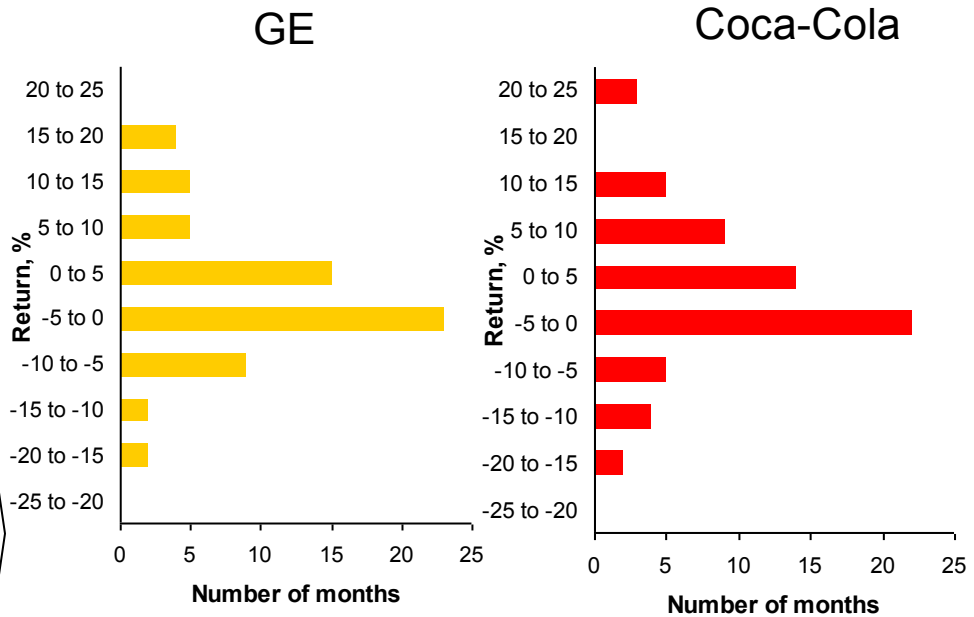
PORTFOLIO 1 WITH THE 3RD ASSET

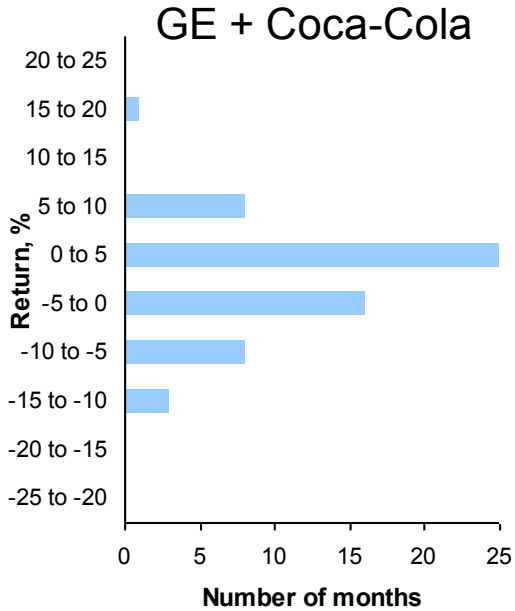
Shares	% of portfolio	Average return	σ
Portfolio 1	50.0%	17.4%	28.1%
Slim Corp	50.0%	19.0%	30.0%
Coefficient correlation		0.3	

- The expected return of the new portfolio is 18.2%
- The standard deviation of the new portfolio is 23.4%
- This shows that the expected return of the portfolio has increased while the portfolio risk is lower than just portfolio 1 or Slim Corp by itself
- This is the effect of diversification

Diversification works because the price of different shares do not move exactly together. In other words, share prices change are less than perfectly correlated

- Examine the outcome of investing all the money in GE only or Coca-Cola only
- Then compare this to the possibility of investing 50/50 in both companies





- If you invest only in GE, there are 4 months when a loss of 10% would occur
- If only invested in Coca-Cola, there are 6 months when a loss of 10% would occur
- In the case of the portfolio, the decline in the value of one share was offset by the other – only in 2 months a loss of 10% would occur and a loss of 15% would never happen

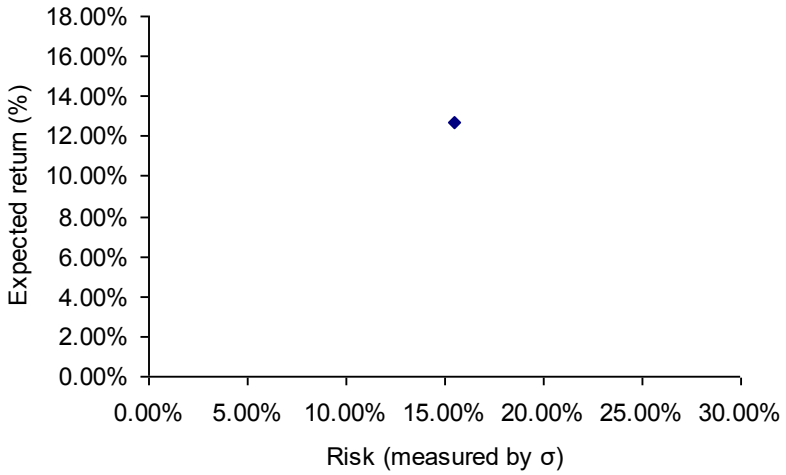
To quantify diversification, let us construct at a 2-asset portfolio

Example

- Two securities have the following statistics
- Let us invest 60% of our investment in security A and 40% in security B
- In this case, expected return is 12.7% and risk is 15.4%

Risk and return characteristics

	Expected Return	σ
Security A	17.50%	25.86%
Security B	5.50%	11.50%
Correlation		-0.1639



- It can be seen that the risk of the portfolio is much lower than Security A and only slightly higher than that of Security B
- The return, on the other hand, is higher than that of Security B and relatively lower than that of A

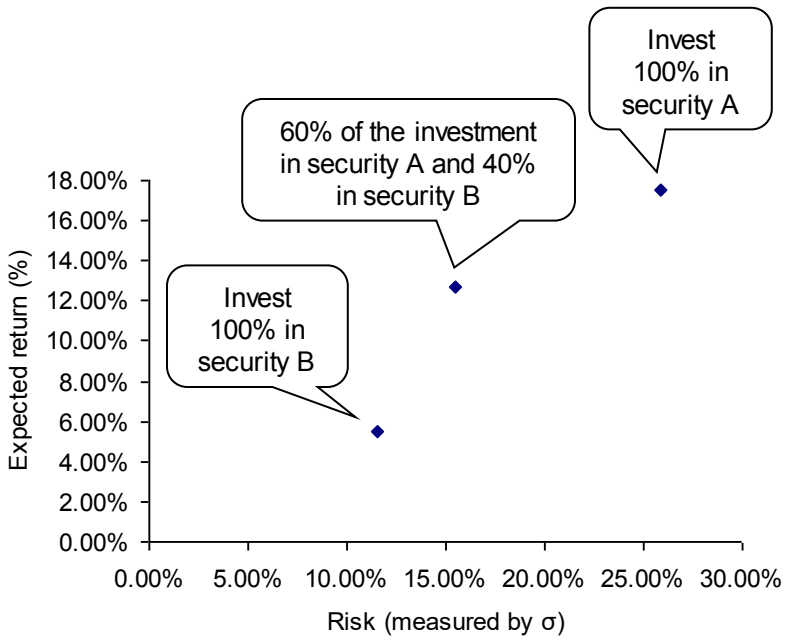
It is possible for investors to have any combination they wish (e.g. 50/50 or 60/40 or 70/30 or 71/29), each of which provides a different risk and return

Example

- Since we can combine 2 assets into a portfolio, there will be a theoretical infinite number of combinations
- The possible combinations are called **opportunity set or feasible set**
- Here, you have 2 securities – A and B, each offering different return and risk
- Depending on the composition, each portfolio made up by the 2 securities provides a different return and risk combination

Risk and return characteristics

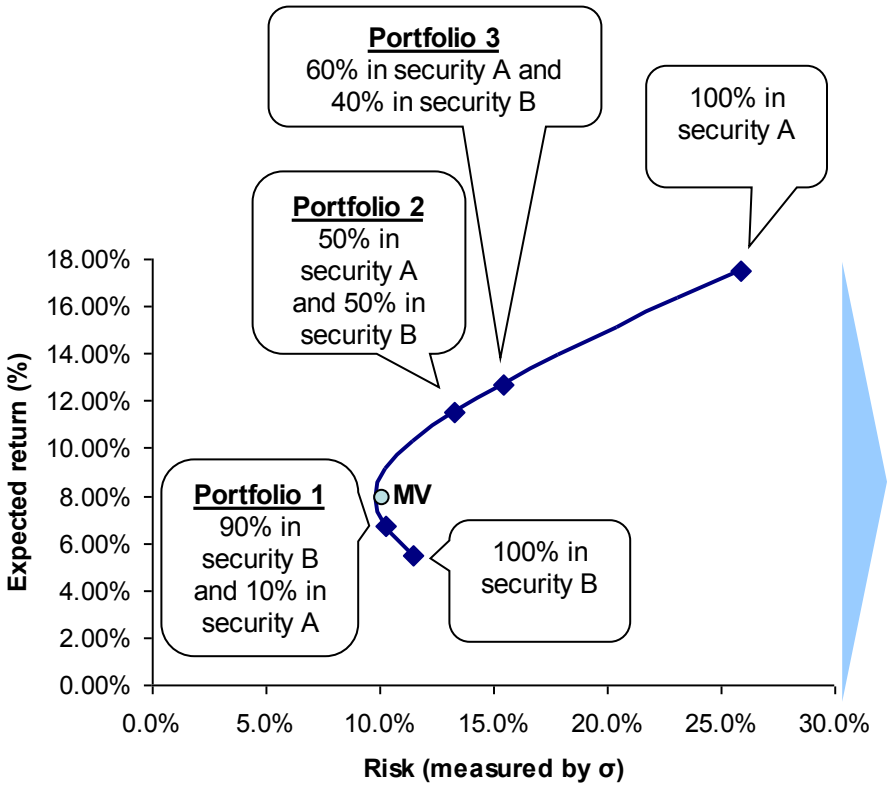
	Expected Return	σ
Security A	17.50%	25.86%
Security B	5.50%	11.50%



- The choice of 60% in security A and 40% in security B is therefore just one of the portfolios that can be created

When deciding an investment in a portfolio made up of shares A and B, investors can choose a portfolio along the curved line that linked all the opportunity sets

- Graphically plotting all the opportunity sets, we can produce a curve
- This curved line shows how expected return and standard deviation change as you hold different combinations of the 2 shares

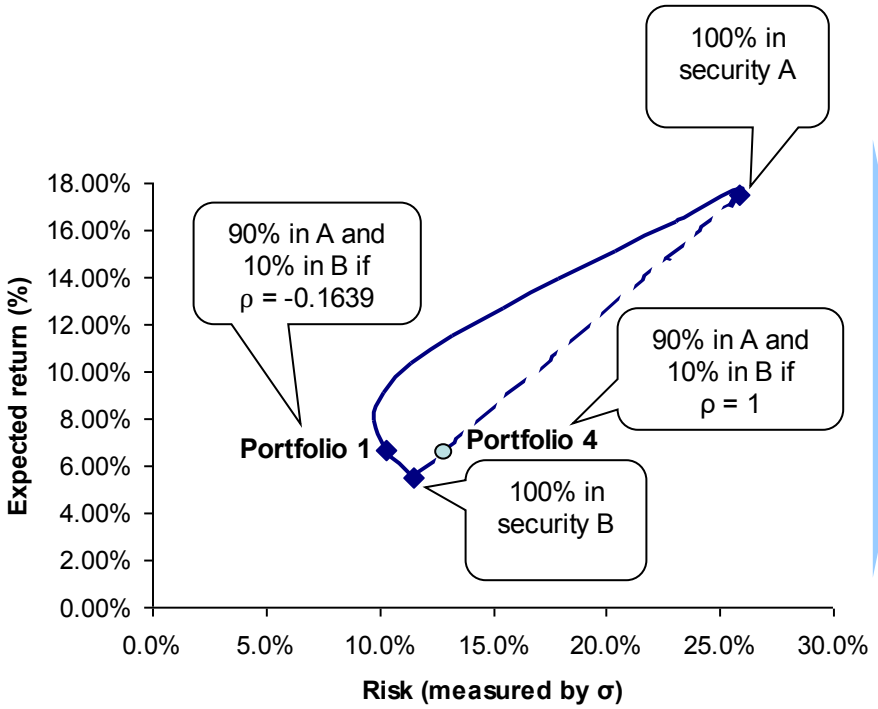


- Portfolios 1, 2 and 3 represent some of the combinations that an investor can have access to
- Of particular interest is the point MV, which stands for the **minimum-variance** portfolio
- By definition, this portfolio has the lowest risk*

** The term 'minimum-variance portfolio' is commonly in use and thus used here. This term can be confusing because it is standard deviation, not variance, measured on the horizontal axis. The term minimum-standard deviation is perhaps more appropriate.*

The curved line also illustrates the effect of diversification

- The diversification effect can be illustrated here with the straight broken line between '100% in security A only' and '100% in security B only'
- The risk and return profiles will be very different if the correlation (or ρ) is different
- For example, Portfolio 1 has the same expected return as portfolio 4, but the standard deviation (σ) is much lower simply because ρ is less than 1



- The fact that the curved line is always to the left of the straight broken line shows the diversification effect

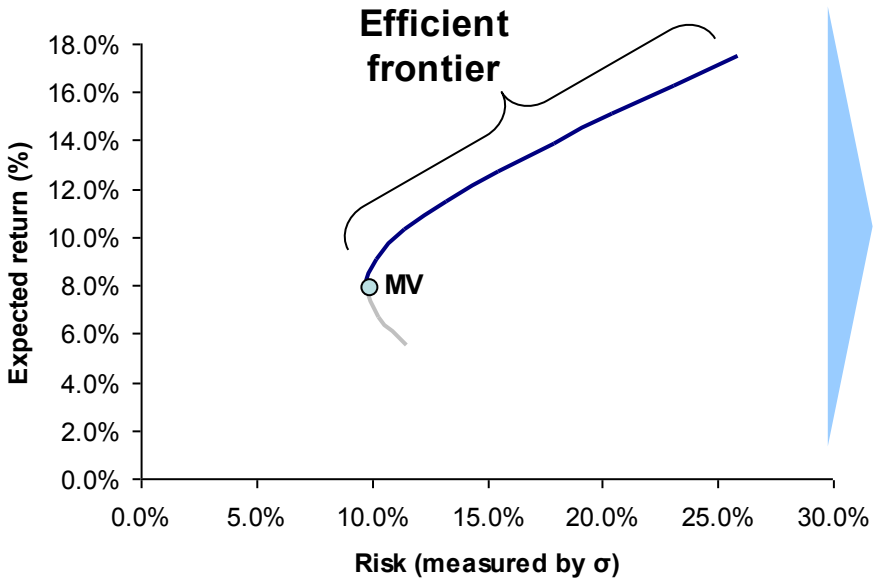
The curved line also illustrates the effect of diversification (cont'd)

- The previous graph and its underlying information reveal the following concepts:

- The straight broken line represents the fact that the correlation between the two securities is equal to 1. By definition, when ρ=1, no diversification can be achieved
- Though the straight broken line and the curved line are both presented in the graph, they do **not** simultaneously exist in the real world. In other words, the correlation between the 2 shares can be **either** $\rho = 1$ **or** $\rho = -0.1639$ and cannot co-exist
- An investor therefore has to choose between different portfolios on the curve if $\rho = -0.1639$ and cannot choose any portfolio on the straight broken line

In fact, this curved line represents all the possible portfolios made up of securities A and B

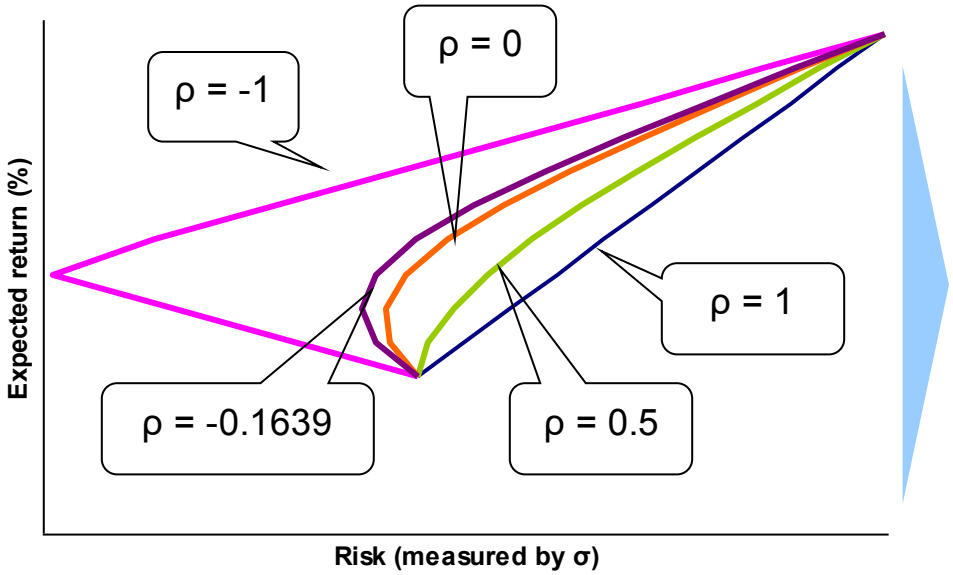
- Since no investor would want to hold a portfolio with an expected return below that of the MV portfolio, we will only consider the part that stretches from MV upwards
- This stretch is called the **efficient frontier**



- It must be noted that investors will not choose any point (or portfolio) *below* this curve because they can neither lower the return, increase the σ of the securities, nor increase the ρ
- Neither can they achieve any point *above* the curve because they can neither increase the return on the individual securities, decrease the σ of the securities, nor decrease the ρ
- On the other hand, what they choose on the efficient frontier depends on how much risk they want to take

As we have seen earlier, correlation plays a crucial role in terms of creating diversification. Indeed, the lower the correlation and therefore the volatility of portfolios, the higher the diversification effect that can be achieved

- The lower the correlation, the more 'bend' the curve will have
- This indicates that the diversification effect works better as ρ declines



- When ρ=1, it is a straight line because the average volatility of the portfolio is equal to the weighed average of the 2 shares
- When ρ = -1 or perfect negative correlation, this line again becomes straight, which makes it possible to hold a portfolio that bears absolutely no risk
- Unfortunately, even negative correlation, let alone perfect negative correlation, is very unlikely

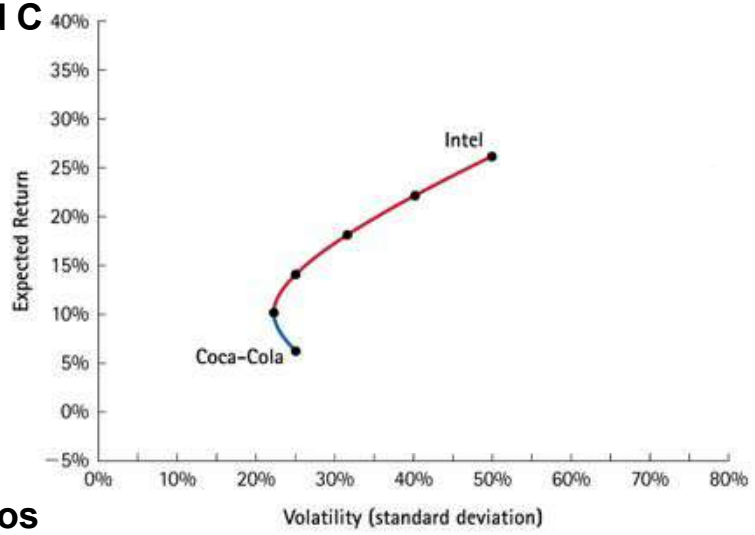
Topic 2: Portfolio Theory

Portfolio with more than two securities

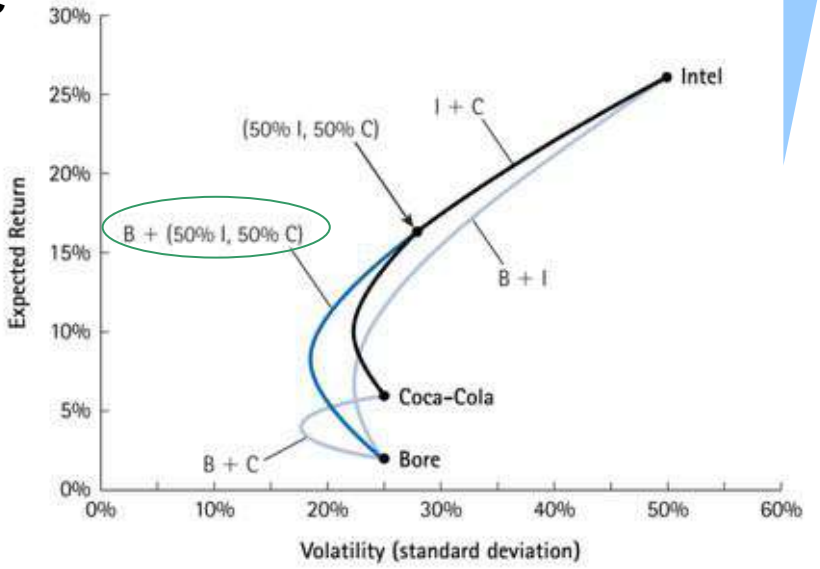
So far we have been concentrating on two securities. In reality, any additional shares in a portfolio will further increase the diversification effect and therefore provides investors with better possibilities of risk and return combination

- With only 2 shares (Intel [I] and Coca-Cola [C]) in a portfolio, an investor can achieve any portfolios on the efficient frontier
- However, with the introduction of an additional share, Bore (B), it brings new possibilities that the combination of I and C cannot previously achieve

Portfolios with I and C



Portfolios with I, C and B

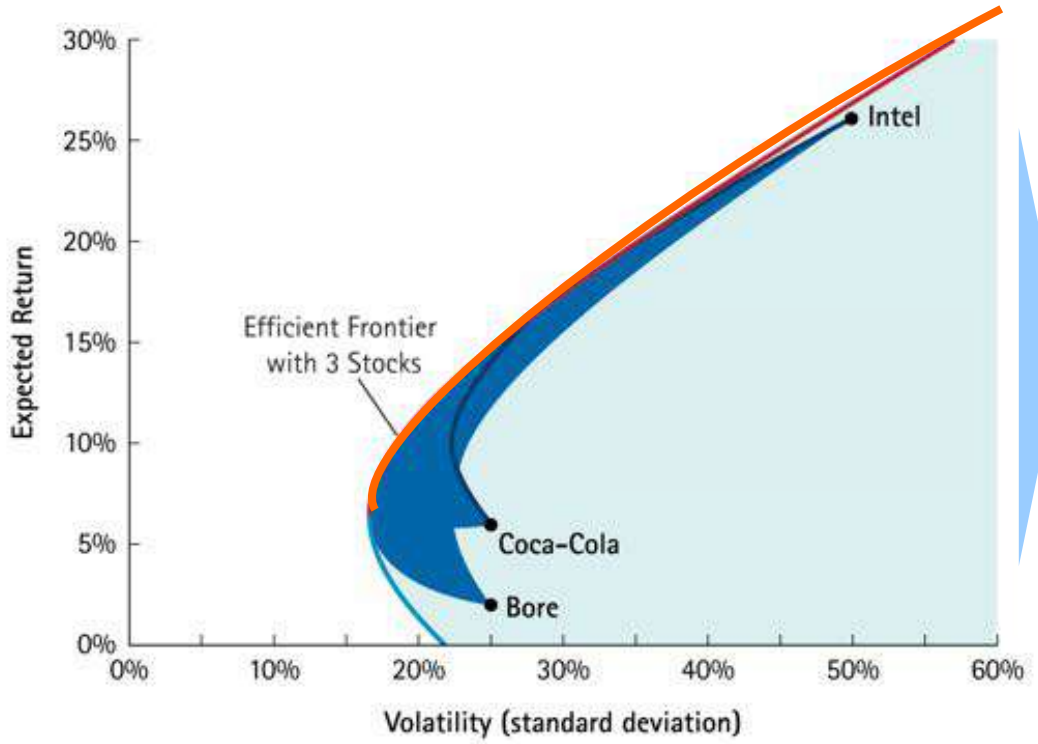


- The inclusion of B in the portfolio ['B + (50% I, 50% C)' in the graph] enables investors to reach the risk and return trade-offs that are not obtainable simply through the combination of 2 securities, whether the combinations are I and C, B and I or B and C

Source: Berk and DeMarzo (2007)

The additional share allows investors to construct portfolios with better risk and return trade-offs

- By joining the curves created by different share combinations, an efficient frontier with 3 shares can be formed (the thick curved line)

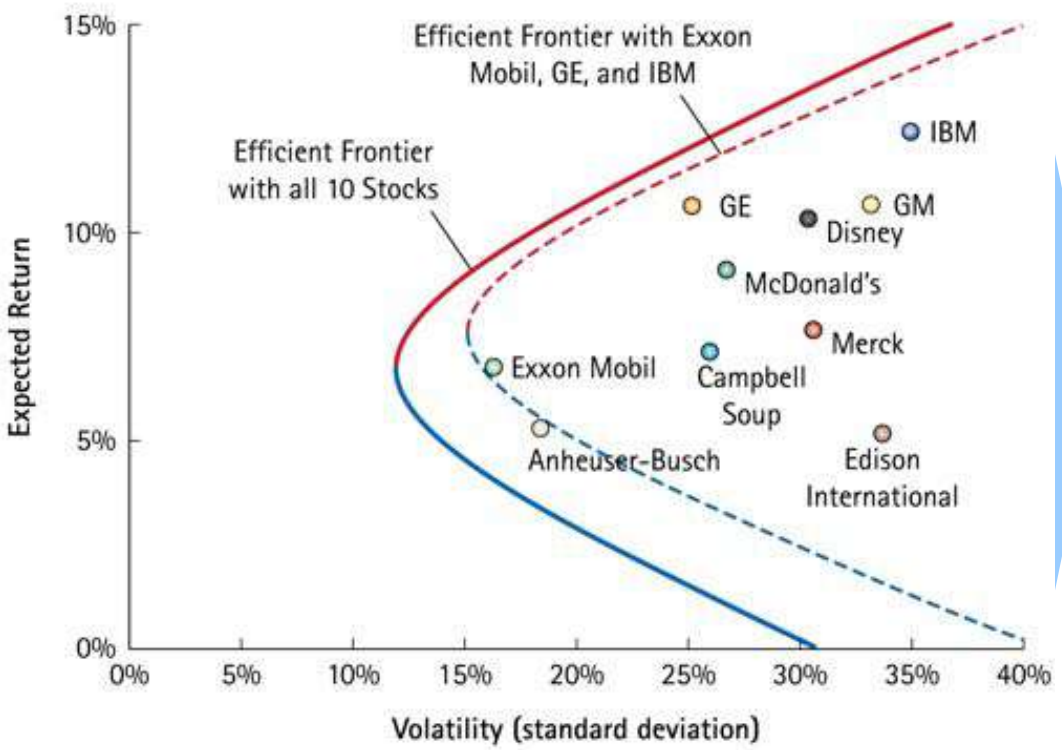


- When only two securities are involved, all the combinations lie on a single curve
- But with many securities, the combinations cover a region (shaded in the graph)
- Thus, the efficient frontier improves (has a higher return for each level of risk) when we move from 2 to 3 shares

Source: Berk and DeMarzo (2007)

As more shares are placed in the portfolio, the efficient frontier can be push further leftward

- It can be seen that a portfolio with 10 shares has lower risk and maintains the same return when compared to a 3-asset portfolio



- Even though the added shares appear to offer inferior risk-return combinations on their own, because they allow for additional diversification, the efficient frontier improves with their inclusion

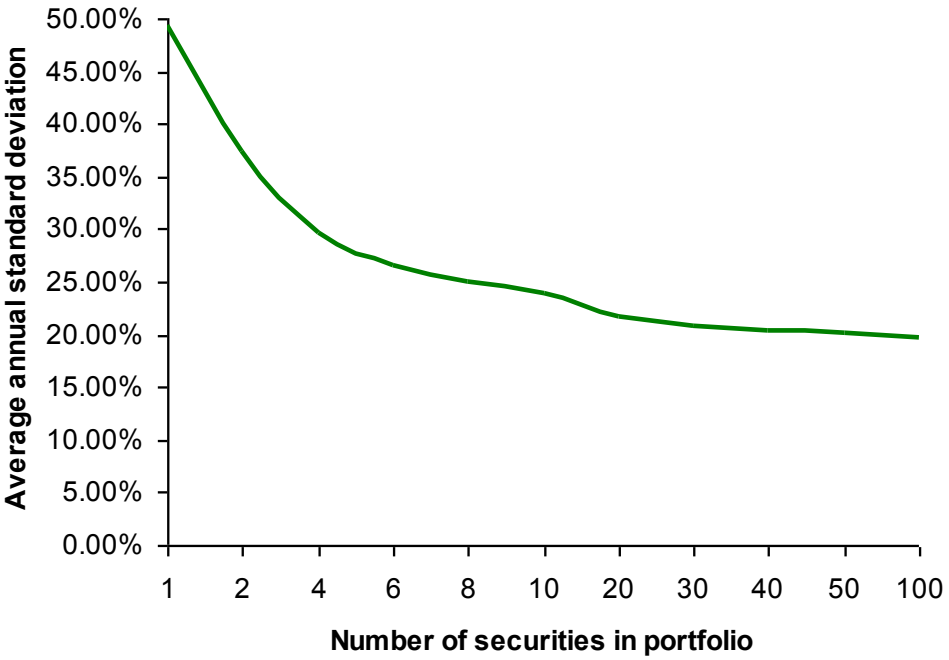
Source: Berk and DeMarzo (2007)

Topic 2: Portfolio Theory

Systematic risk and non-systematic risk

We have seen the effect of diversification. Does that mean we can keep on adding shares to diversify away all the risk? The answer is that risk will continue to reduce with each share added to the portfolio – but only up to a certain point

- Using the average annual σ of portfolios containing different numbers of randomly selected securities on the NYSE, it can be seen that each additional security in the portfolio can lower its risk, with the σ declining as the number of securities is increased

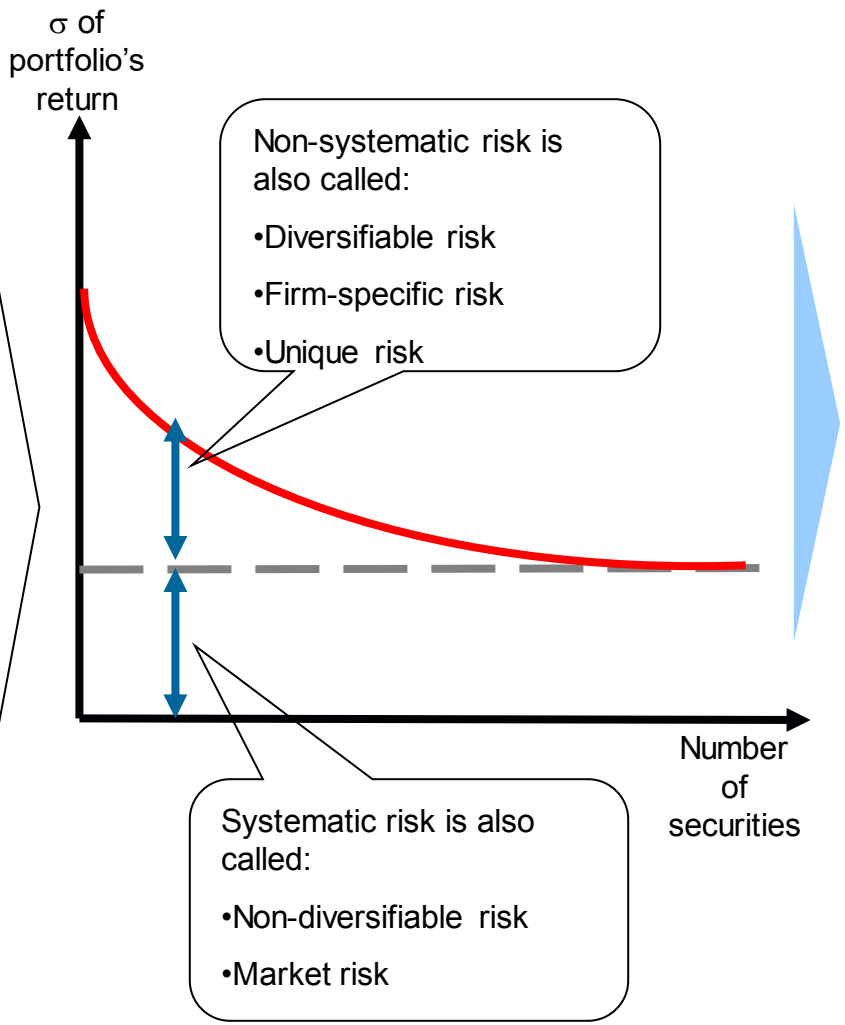


- However, the decrease in risk gradually levels off even though more and more securities are added into the portfolio
- This shows that we can only go so far with diversification

Source: Statman, Meir (1987) "How many stocks make a diversified portfolio?" Journal of Financial and Quantitative Analysis 22, pp. 353-364

In other words, there is risk that can be diversified away and there is risk that cannot be diversified away

- The risk that cannot be eliminated is called **systematic risk,** while the risk which can be eliminated is called **non-systematic risk**



- We therefore have two types of risk:

Non-systematic risk

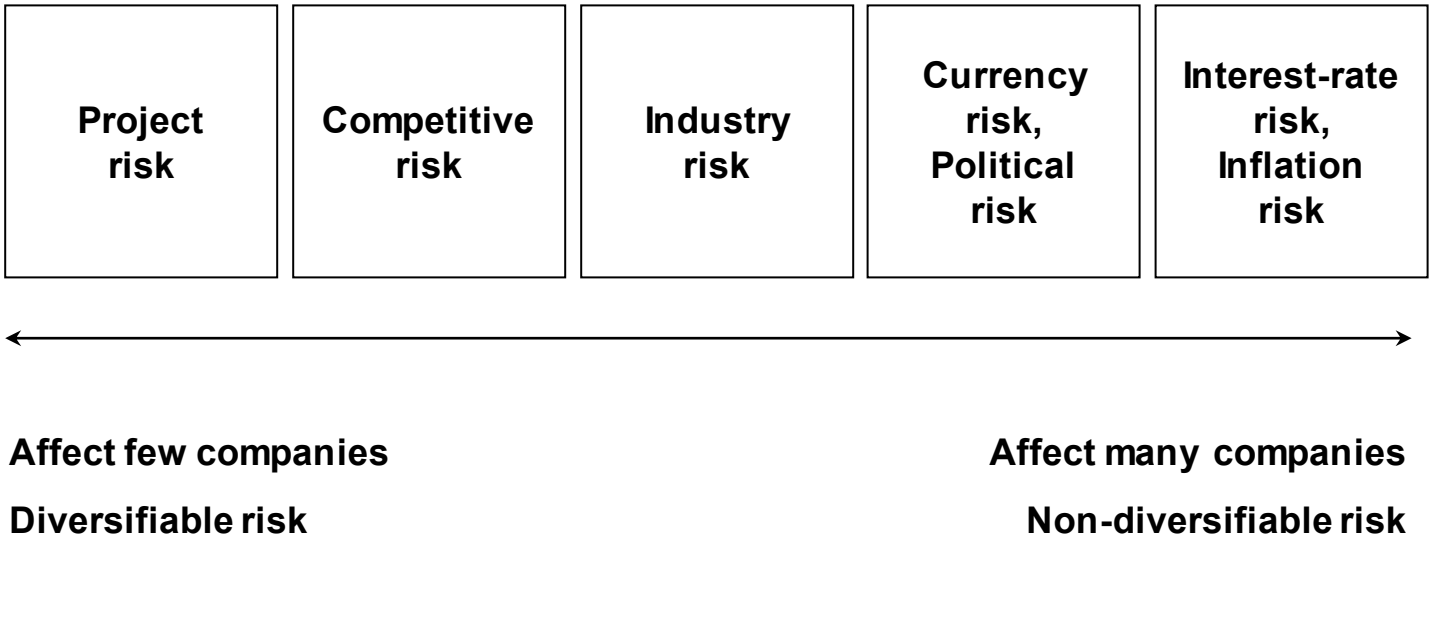
- Risk that can be eliminated by diversification (hence 'diversifiable' risk)
- Risk that affects at most a small number of assets (hence 'firm-specific' risk)
- Component of total risk which is unique to an asset or firm (hence 'unique' risk)

Systematic risk

- A risk that influences a large number of assets (hence 'systematic' risk)
- Component of total risk which is due to economy-wide factors (hence 'market risk')
- Cannot be eliminated by diversification (hence 'non-diversifiable' risk)

In other words, there is risk that can be diversified away and there is risk that cannot be diversified away (cont'd)

- Companies face different types of risk and some are diversifiable while others are not



In other words, there is risk that can be diversified away and there is risk that cannot be diversified away (cont'd)

- From the previous graph, it can be seen that

- Total risk any investor faces is equal to the sum of systematic risk and unsystematic risk
- While it is possible to diversify the risk related to the firms, it is impossible to get rid of the risk inherent to the market

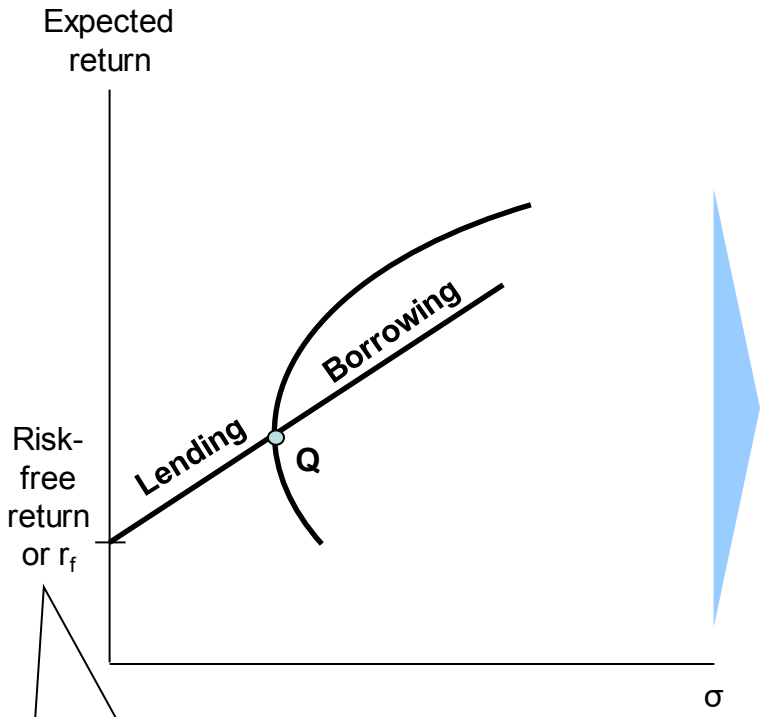
- Therefore, it can be concluded that:
  - Since non-systematic risk has been eliminated by diversification, a portfolio that is relatively large in size will have almost no non-systematic risk
  - Extending this logic, the expected return on a risky portfolio is due to the systematic risk

Topic 2: Portfolio Theory

Portfolios with risk-free borrowing and lending

Even though the region above the efficient frontier cannot be achieved by trading risky assets, it is reachable through holding risk-free assets

- So far, we have concentrated on combining risky investments into portfolios to create an efficient frontier so as to achieve diversification
- But one can also diversify by including no-risk investments like US Treasury bills and UK government bonds
- Conversely, we can borrow to seek higher returns
- Let us pick a portfolio on the efficient frontier at random: point Q, which represents a portfolio of securities with no risk-free assets included



- It is risk-free because it has zero variance
- **It is also uncorrelated with any other asset (since its variance is zero)**

Lending = investing in Q and the risk-free asset

Borrowing = investing in Q by borrowing money at risk-free rate

- If you invest some money (i.e. lend) in a risk-free asset and place the remainder in portfolio Q, you can obtain any combination of expected return and risk along the straight line joining r_f and Q (called 'Lending')
- Since borrowing is merely negative lending, you can extend the range of possibilities to the right of Q by borrowing funds at an interest rate of r_f and investing them as well as your own money in portfolio Q (called 'Borrowing')

Even though the region above the efficient frontier cannot be achieved by trading risky assets, it is reachable through holding risk-free assets (cont'd)

- Let us examine the return and risk that we can get with access to borrowing and lending at risk-free rate
- R_{XQ} refers to the portfolio with risk-free assets

Expected return

$$E(R_{XQ}) = (1-a)r_f + aE(R_Q)$$
$$= r_f - ar_f + aE(R_Q)$$
$$= r_f + aE(R_Q) - ar_f$$
$$= r_f + a[E(R_Q) - r_f]$$

This bit is called risk premium because this rewards the investors who are willing to invest in non-risk-free assets

R_Q = A risky portfolio
a = Proportion of money in the portfolio
(1-a) = Remaining portion of money in the portfolio
r_f = risk-free assets

Risk

$$\sigma(R_{xQ}) = \sqrt{(1-a)^2\sigma_{r_f}^{\ 2} + a^2\sigma_{R_Q}^{\ 2} + 2(1-a)(a)\rho_{r_f,R_Q}\sigma_{r_f}\sigma_{R_Q}}$$

Since r_f is risk-free investment, risk is equal to zero,

$$= \sqrt{a^2\sigma_{R_Q}^{\ 2}}$$
$$= a\sigma_{R_Q}$$

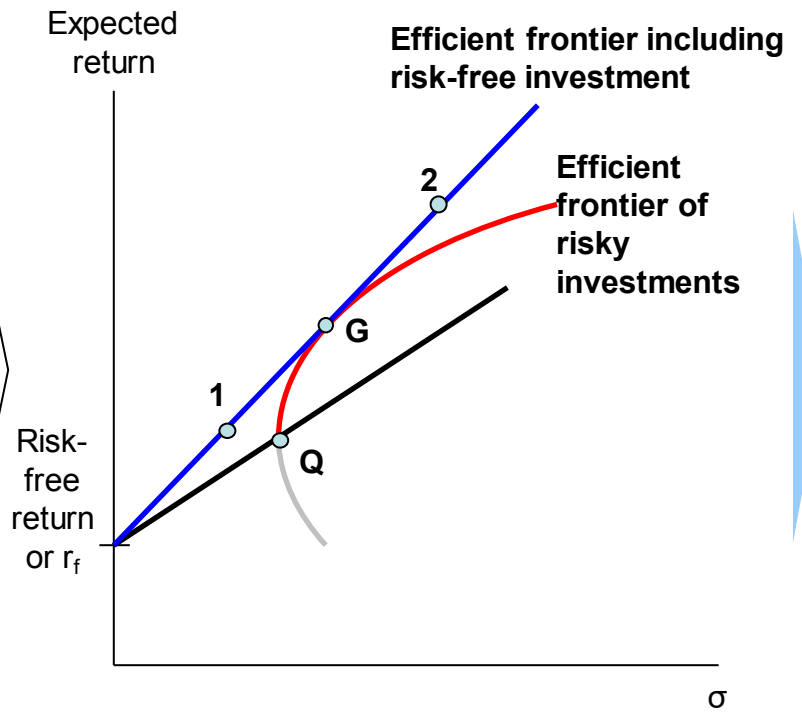
This is exactly the same formula you saw in the previous lecture:

$$\sigma_P^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho_{XY}\sigma_X\sigma_Y$$

- Expected return is equal to risk-free rate plus a fraction of the risk premium of the portfolio based on the amount invested
- Risk, on the other hand, equals to the fraction of the risk of the portfolio based on the amount invested
- If you plot the graph for the combinations of expected return and risk, you will get the straight line (the one that goes through Q in the previous graph)

A new efficient frontier can be established as a result of including risk-free assets

- But portfolio Q in the previous graph is not the best portfolio to combine with the risk-free investment
- By forming a portfolio out of the risk-free asset and portfolio somewhat higher on the efficient frontier than portfolio Q, we will get a steeper line
- If the line is steeper, for any level of risk, it is possible to gain a higher return



- G is called the **efficient** (or **tangent**) **portfolio** as it provides the best risk and return trade-off when risk-free borrowing and lending is accessible
- It is necessary to note that the inclusion of risk-free asset changes the return and risk relationship (and hence the efficient frontier) from a curve into a straight line

A new efficient frontier can be established as a result of including risk-free assets (cont'd)

- The previous graphs and their underlying information reveal the following concepts:

- Lending or borrowing at risk-free rate allows us to go beyond the efficient frontier. In other words, investors can add leverage to the portfolio by borrowing the risk-free asset. The combination of the risk-free asset with risky assets enables the creation of portfolios with risk-return portfolios that are superior to those efficient frontiers with only risky investments
- Investors with a higher degree of risk aversion might combine G with an investment in the risk-free asset, achieving, say, point 1 in the previous graph. In short, these investors are **de-leveraging** their portfolios by holding a portfolio of risk assets with a simultaneous holding in cash
- Other investors with low aversion to risk (i.e. more risk-seeking) might borrow to achieve, say, point 2. In a nutshell, these investors have a **leveraged** portfolio as they borrow money to fund the purchase of risky assets

In order to determine the efficient frontier including risk-free investment, it is necessary to calculate the Sharpe ratio

- What is the best portfolio when we have access to both risky and risk-free assets?

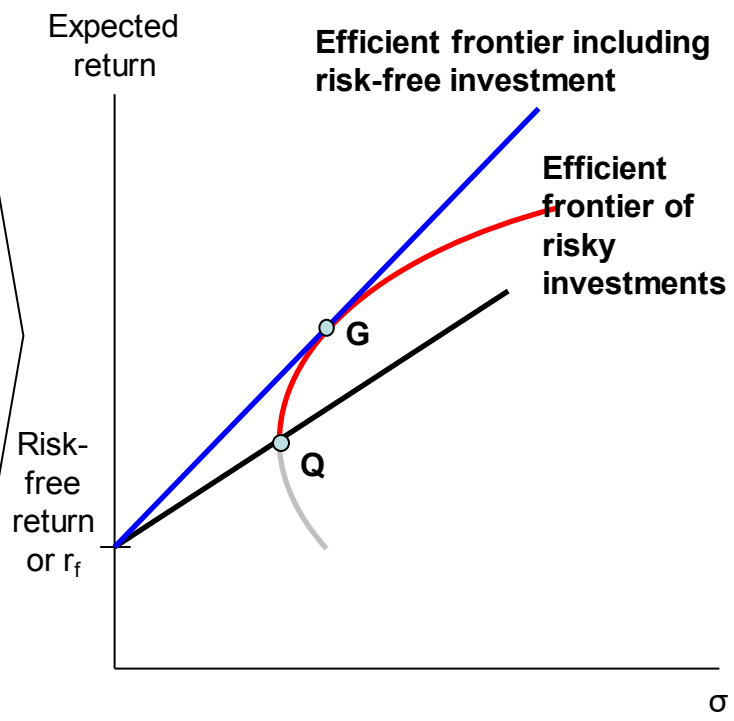
- In other words, what is the efficient portfolio, the one which refers to the highest possible return for any level of risk? In yet other words, how can we determine G?
- To identify the efficient portfolio, we must find the portfolio that has the steepest possible line when combined with the risk-free investment. The slope of the line through a portfolio is often referred to as the **Sharpe ratio** of the portfolio:

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Excess Return}}{\text{Portfolio Risk}} = \frac{E(R_P) - r_f}{\sigma(R_P)}$$

- The ratio therefore describes the best risk-return trade-off. You can also think of it as the ratio that measures the ratio of reward to risk provided by a portfolio

With the Sharpe ratio, it is possible to determine the efficient portfolio (or tangent portfolio)

- By calculating the highest Sharpe Ratio, it is possible to determine the efficient portfolio on efficient frontier of risk investments (point G)



- By joining the risk free investment and the efficient portfolio (point G), we can produce an efficient frontier including risk-free investment
- It is important to note that portfolio G represents the only portfolio on the efficient frontier including risk-free investment that contains only risky assets and no risk-free assets
- All other portfolios on the frontier must include risk-free assets to be achieved. In other words, no other portfolio that consists of only risky assets is efficient
- This is an important conclusion because the optimal portfolio of *risky* investments no longer depend on the investors' tolerance of risk. Instead, their preference will determine only **how much** to invest in risk-free asset

Example

- Assume that there are 2 assets, a risky investment called Umbrella Corp. and a risk-free asset in the form of a bank account
- Each has the following risk and return characteristics
- It is necessary to note that there are 2 possible states of outcome – sunny or rainy – for Umbrella Corp. in terms of return
- The probability of these outcomes is assumed to be 50-50

	Expected Return		**Mean expected return**	**σ**
	Sunny	**Rainy**		
Umbrella Corp.	-10%	30%	10.0%	20%
Bank account (risk-free)	3%	3%	3.0%	0%

Example (cont'd)

- Let's say we have £100 to invest
- If we invest £50 in Umbrella Corp. and the remaining £50 in the bank account, we will get an investment with the following characteristics

Scenario 1 (lending)

	Capital invested	Expected Return		Mean expected return		σ
		Sunny	Rainy	Amount	%	
Umbrella Corp.	50	45	65	55	10%	20%
Bank account (risk-free)	50	51.5	51.5	51.5	3%	0%
Portfolio	**100**	**96.5**	**116.5**	**106.5**	**6.5%**	**10.0%**

- By investing a portion of the risk-free asset, the mean expected return will drop…
- …but not to the same extent as the level of risk decreases

Example (cont'd)

- Let us now look at the case in which we borrow £50 from the bank and invest all in Umbrella Corp.
- We can get the following

Scenario 2 (borrowing)

	Capital invested	Expected Return		Mean expected return		σ
		Sunny	Rainy	Amount	%	
Umbrella Corp.	150	135	195	165	10%	20%
Bank account (risk-free)	(50)	(51.5)	(51.5)	(51.5)	3%	0%
Portfolio	**100**	**83.5**	**143.5**	**113.5**	**13.5%**	**30.0%**

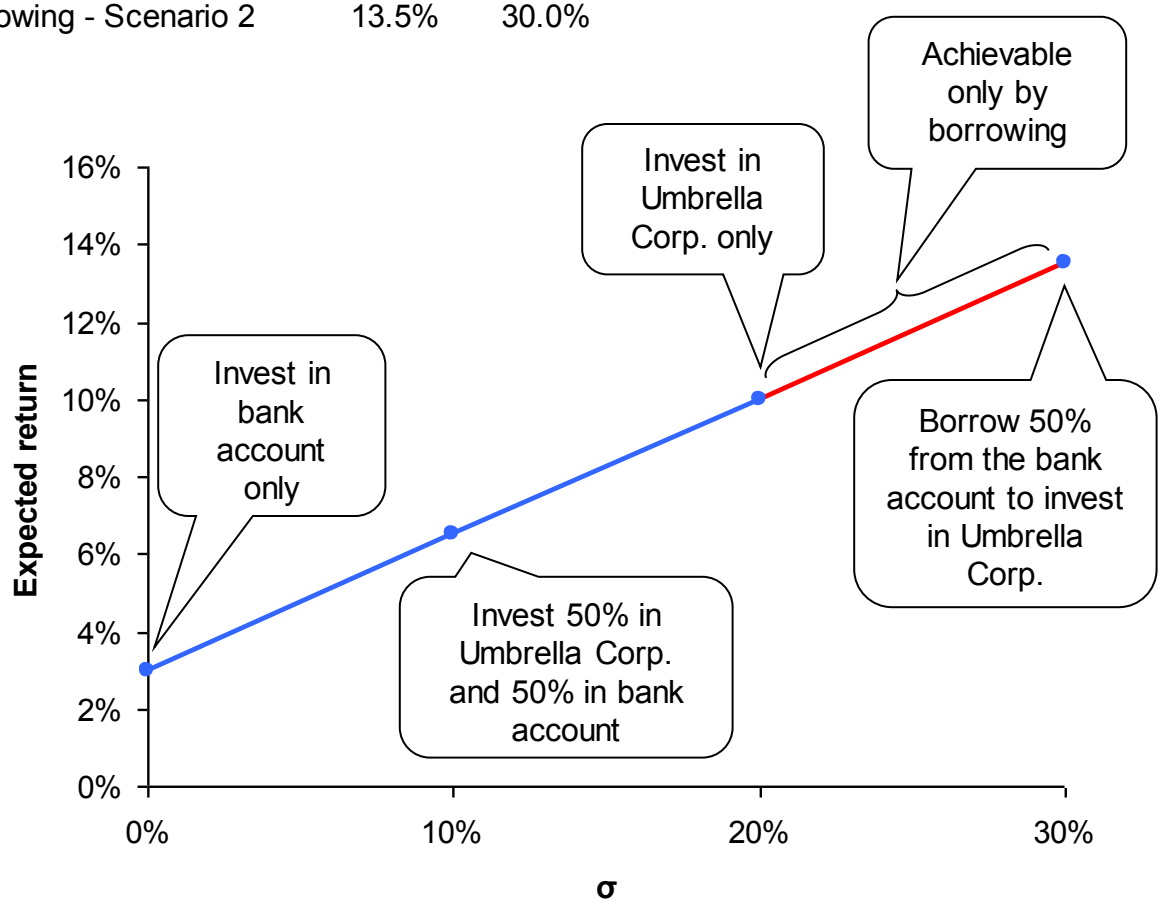
- With such financial leverage, investors can potentially gain more when the state is in their favour but lose more when the state is against them

Example (cont'd)

SUMMARY

	Expected return	σ
Umbrella Corp.	10.0%	20.0%
Bank account (risk-free)	3.0%	0.0%
Lending - Scenario 1	6.5%	10.0%
Borrowing - Scenario 2	13.5%	30.0%

- Graphically, we can see all the possible risk and return of the different portfolios

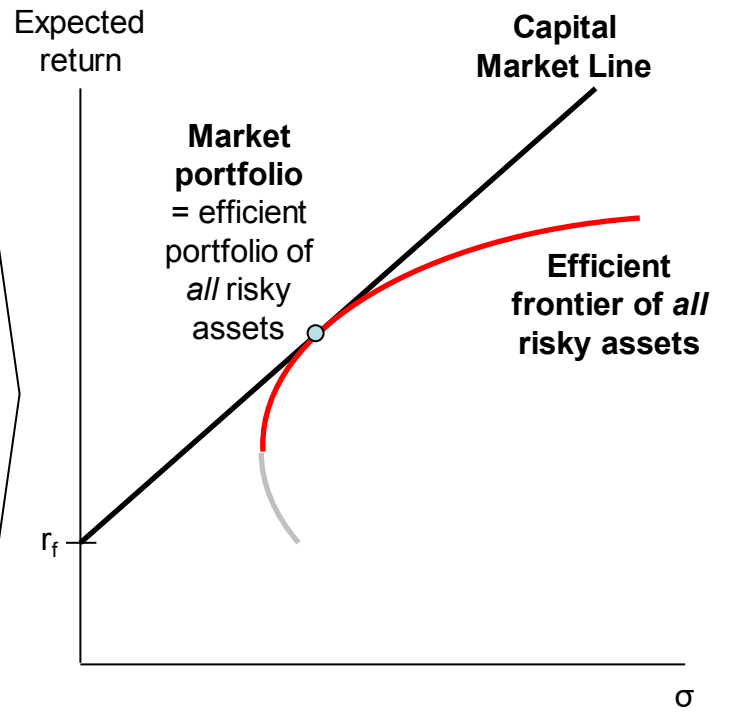


Topic 2: Portfolio Theory

Market portfolios and capital market line

When we aggregate *all* the risky securities in the market, we can determine the market portfolio…

- So far, we have only considered individual investors
- But different investors can obviously have different estimates of expected returns, variances and covariances
- Yet, it has been argued that they should not vary a great deal because all investors would be forming expectations from the same data about past price movements and other publicly available information
- Hence, everyone will have **homogenous expectations**
- If all investors choose the same portfolio of risky assets, the efficient portfolio can be seen as the **market portfolio**



- The point at which the line is tangent to the efficient frontier is the market portfolio
- The market portfolio corresponds to the portfolio made up of all the shares trading on the stock market
- Hence, the market returns can be estimated by the returns on market index

…as well as the capital market line

- When the market portfolio is combined with the risk-free asset, the efficient frontier is called the **capital market line** (CML), as shown in the previous graph

- In the presence of risk-free assets, the efficient portfolios are on the CML
- It represents the highest expected return available for any level of volatility
- All points along the CML have superior risk-return profiles to any portfolio on the efficient frontier of all risky securities, except where market portfolio is
- Any rational investor will thus choose a combination of risk-free asset and market portfolio
- To get a higher return than the market, the investor must borrow at risk-free rate and invest the total amount in the market portfolio

Topic 2: Portfolio Theory

Beta

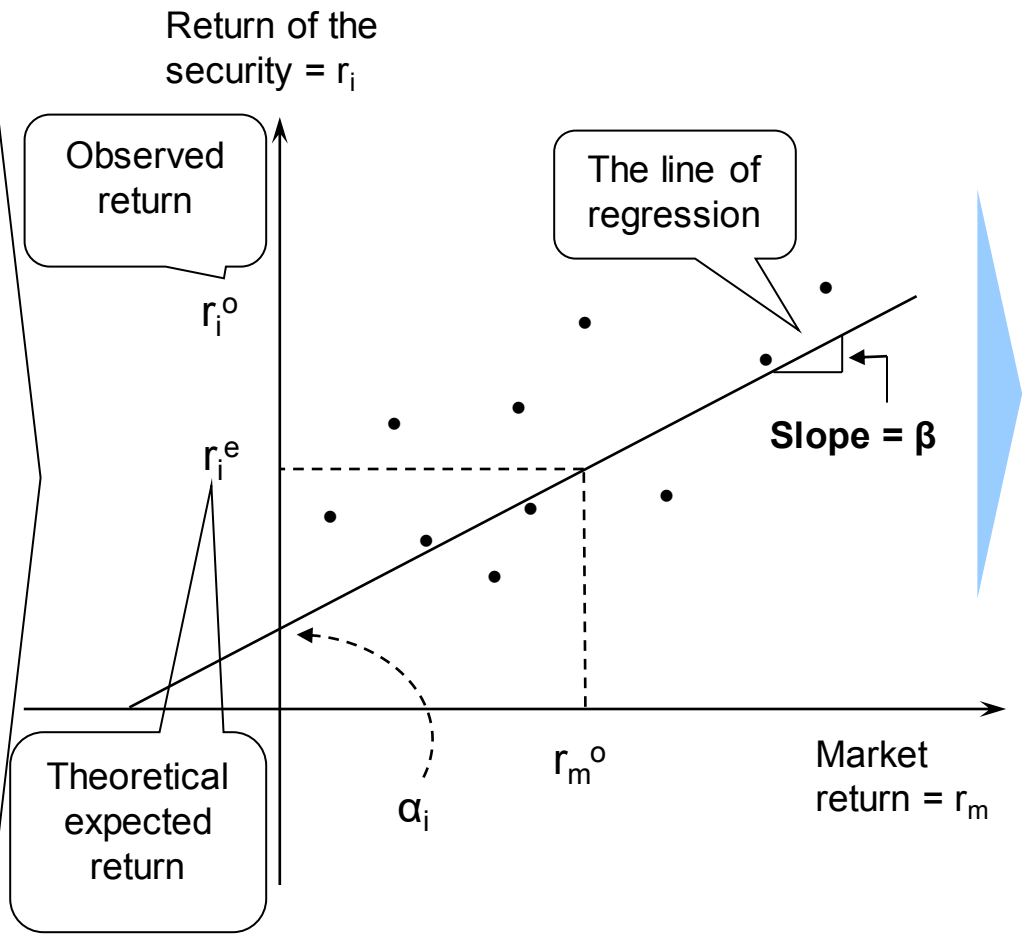
Earlier, it was shown that a substantial amount of the risk in a portfolio (measured in σ) can be eliminated, while the rest cannot be diversified away

Types of risk	Description
Non-systematic risk	• This is the part of a portfolio's risk that can be eliminated by holding a high number of shares • Therefore, in fully diversified portfolio, there will be *no more* non-systematic risk
Systematic risk	• There are also risks that we cannot eliminate by adding shares through diversification; the variability is caused by events that affect most shares simultaneously • Consequently, no matter how much total risk an asset has, only the systematic portion is relevant in determining the expected return • So, when considering the level of risk of an asset, we can very much concentrate on systematic risk as any risk unique to the assets can be diversified away

- Many investors hold diversified portfolios similar to broad-based indices such as S&P 500
- Since systematic risk is the crucial determinant of an asset's expected return, we need some way of measuring the level of systematic risk for different investments

The specific measure used for assessing the level of systematic risk is called the beta coefficient (or β)

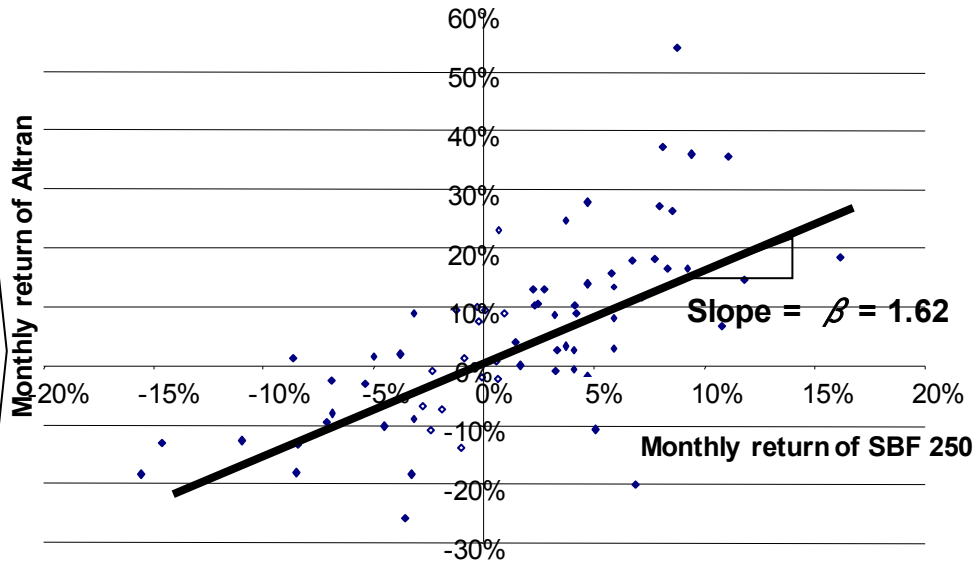
- β describes how the expected return of a security is correlated to the return of the financial market as a whole
- Therefore, it describes the systematic risk of a security
- β is derived by regressing a set of observations on the possible returns, both of a security and of the market



- β measures the sensitivity of a security to the fluctuations of the market
- The regression is: $r_i^o = \alpha_i + \beta_i r_m$
- Since α_i is 'specific' and 'unique' to investors and unpredictable, the return for the systematic risk is $\beta_i r_m$
- Hence, the expected return on an asset is β multiplied by the market return

The specific measure used for assessing the level of systematic risk is called the beta coefficient (or β) (cont'd)

- Here is an example of calculating the beta of the share of Altran Technologies between 1996 and 2002
- By drawing the regression line through the observed return combinations, the β of the company can be established



Source : Christoph Thibierge

- A β of 1.62 means that the return of Altran can be expected to be 1.62 times the *change* of the market return
- In other words, if the market return goes up by 1% Altran's return can be expected to increase by 1.62%
- The same applies to the opposite – ⇩ market 1%, ⇩ Altran 1.62%

Knowing the β of an asset allows us to assess the sensitivity of the asset to the market

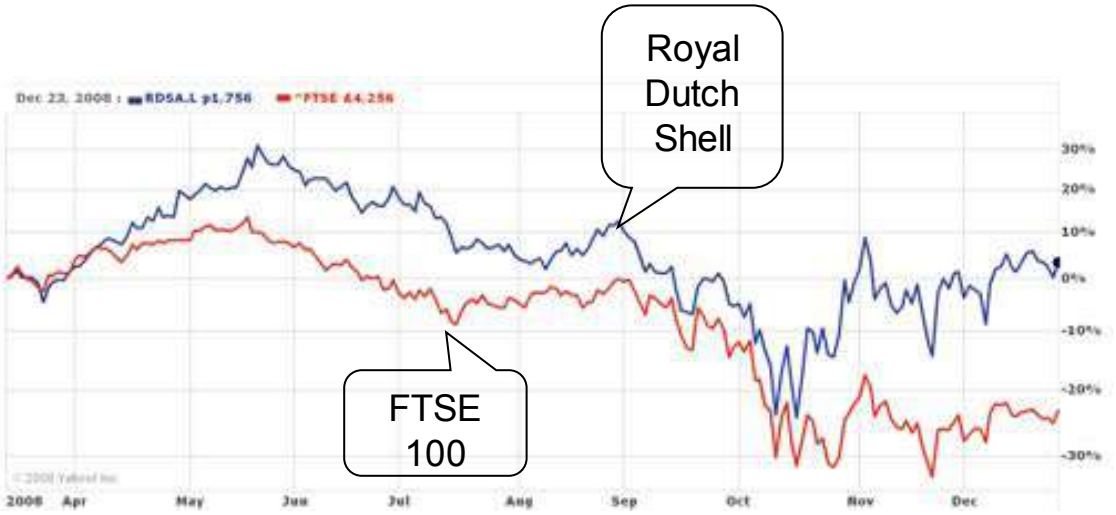
- By definition, the market β is equal to 1
- Unless a security moves in tandem with the market (i.e. its β is also 1), the β is likely to be higher or lower than 1
- This gives us insight into this sensitivity

β	Related stock price evolution
Above or well above 1	• The related stocks are cyclical and volatile racers • Fast climbers and fast divers • More volatile than the market
Close to 1	• Foot soldiers marching in sync with the market • Share prices move like the whole group average price
Between 0 and 1	• Less volatile than the market • Defensive stocks that are dull, not very profitable • However, they are safe investments
Equal to 0	• Theoretically, a return-less asset
Less than 0	• Dissident stocks which prices are inversely correlated to the whole market • When the index makes a zig their price makes a zag, and vice versa

- Assets with larger βs have greater systematic risks but investors in those assets will receive higher returns

Firms with a β close to 1 can be expected to follow a very similar movement as the market

- Let us compare Royal Dutch Shell (which has a beta of 1.02) to the FTSE 100



- They both demonstrate very similar ups and downs

Source: Yahoo! Finance

Firms with a β greater than 1 can be expected to follow an amplified movement of the market

- Let us compare Rio Tinto (which has a beta of 1.89) to NYSE

- Rio Tinto's shares magnify market fluctuations whether the latter moves up or down

Source: Yahoo! Finance

Firms with a β less than 1 can be expected to follow the smaller movement compared to that of the market

- Let us compare RWE (which has a beta of 0.66) to DAX

- RWE displays less sensitivity to the market
- It is less affected by market fluctuations

Source: Yahoo! Finance

In addition to performing a regression, it is also possible to calculate β by covariance of the returns on the security and the market and variance of the market return

- Since we can derive β by performing a regression of security vs. market returns, we can see that the β of security i in relation to the market (m) is:

$$\beta_i = \frac{Cov(i,m)}{\sigma_m^2}$$

$$= \frac{\rho_{im}\sigma_i\sigma_m}{\sigma_m^2}$$

$$= \frac{\rho_{im}\sigma_i}{\sigma_m}$$

Systematic risk of security i

Total risk of the market portfolio (which is all systematic risk)

- Hence, the β value of security i is the systematic risk of security i related to the total market risk

Topic 3: CAPM

CAPM

If all the unique risk can be diversified away, the return that investors can obtain must be related to the systematic risk only

- In other words, investors would only be compensated for the market risk and not the diversifiable risk
- The expected return of security i can be represented by:

$$r_i = r_f + \text{‘Risk Premium’}$$

- The expected return on the security is the sum of the risk-free rate plus some compensation for the risk inherent to the security
- Because shares have risk, the actual return on the market over a particular period can of course be below r_f or even negative
- However, since investors want compensation for risk, the risk premium should be positive
- The risk premium should reflect the difference between the historical return of the market and the risk-free rate

To determine the expected return of an asset and the risk premium, we can use the capital asset pricing model, or CAPM

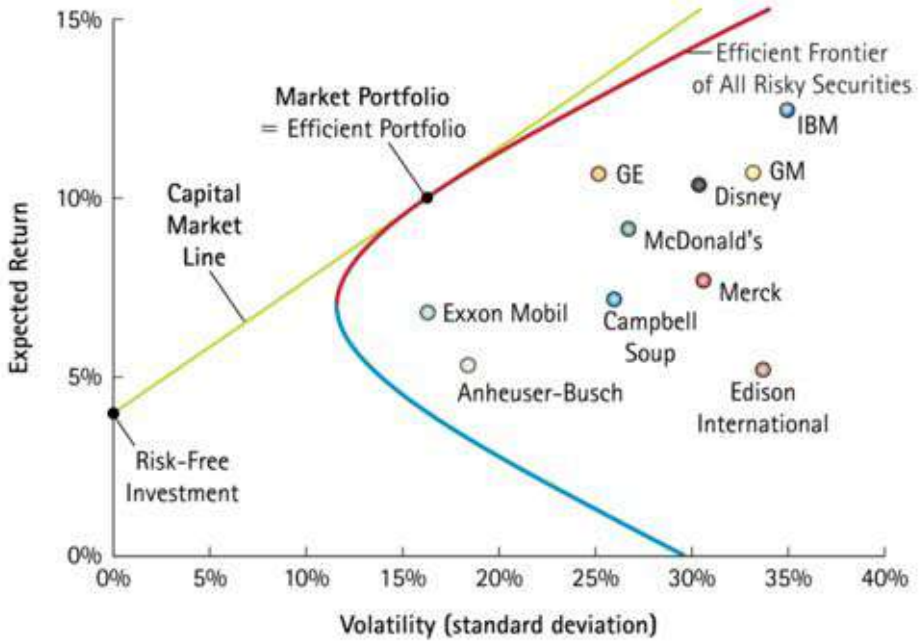
- In the early 1960s, Sharpe, Treynor and Lintner developed an asset pricing model that measures only the systematic risk of a particular asset
- The reasoning behind this is that the risk premium on a particular asset should be determined by the level of its systematic risk as all the non-systematic risk can be diversified away
- This model, called the CAPM, is based on the principle of market equilibrium with 3 major assumptions made:

Assumptions

1. Investors can buy and sell at competitive market prices (without incurring taxes and transaction costs), and can borrow and lend at the risk-free interest rate
2. Investors hold only efficient portfolios of traded securities, that is portfolios that yield maximum expected return for a given level of volatility
3. Investors have **homogenous expectations** regarding the risks, correlations and expected returns. Homogenous expectations refer to the (imagined) possibility of a world where all the investors possess the same estimates of expected returns, variances and co-variances. The result is that everyone will hold the same portfolio

If systematic risk is the only risk that is compensated and all investors hold the same portfolio, then it is necessary to find out what this very portfolio is

- From the previous session, it was shown that when investors have homogenous expectations, the market portfolio and the efficient portfolio will be the same
- Therefore, the CML represents the highest expected return available for any level of volatility

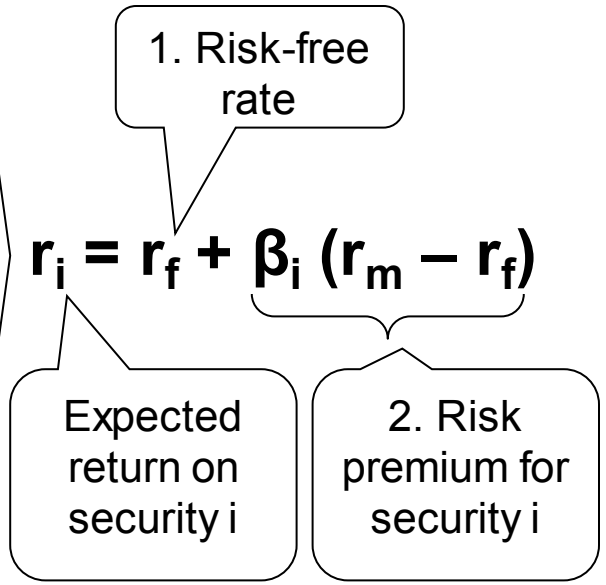


Source: Berk and DeMarzo (2007)

- If the efficient portfolio is the market portfolio, we can determine the expected return for a security through the use of the market portfolio as a benchmark
- In other words, the return of the market portfolio can be seen as market return

With the use of risk-free rate, the β of the security and the market returns, we can use the CAPM to assess the expected return of the security

- The CAPM states that the expected return on asset (could be an individual security or a portfolio of assets) is determined by
  1. the risk-free rate (return that an investor can get without taking any risk); and
  2. a risk premium (which can be determined by the beta coefficient of the asset and the market risk premium [the reward for bearing the systematic risk])



- Therefore, the risk premium of a security is equal to the market risk premium (the amount by which the market's expected return exceeds the risk-free rate) [r_m-r_f], multiplied by the amount of market risk present in the security's returns [β]
- Note that ideally, we would have the expected return of the efficient portfolio as r_m
- However, finding the expected return of the efficient portfolio requires information about all securities' expected returns, risks and correlations, which is time-consuming if not impossible

With the use of risk-free rate, the β of the security and the market returns, we can use the CAPM to assess the expected return of the security (cont'd)

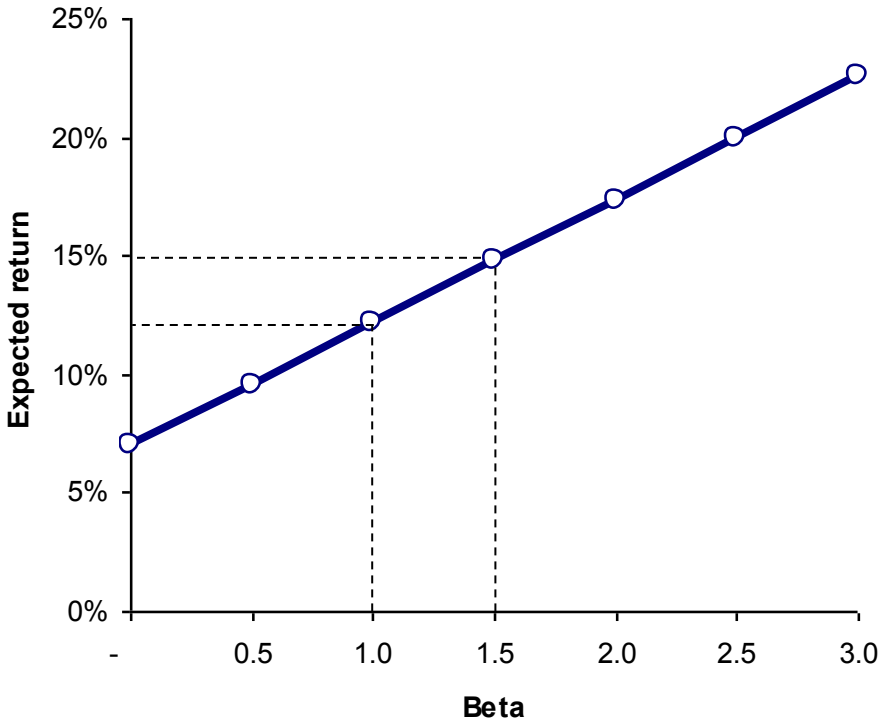
- Additional insights from the CAPM

- The β of a security is the ratio of its volatility due to market risk to the volatility of the market as a whole
- CAPM implies a linear relationship between a share's β and its expected return
- Because the average return on the market has been higher than the average risk-free rate over long periods of time, $r_m - r_f$ is presumably positive
- Thus, the formula implies that the expected return on a security is positively related to its beta
- The CAPM says that if all investors hold the market portfolio, the risk premium they will demand is proportional to the market β

- Once again, in order to minimise total risk, investors seek to reduce the component which can be reduced, i.e. the specific risk. They do so by diversifying their portfolios. As a result, when stocks are fairly valued, investors will receive a return only on the portion of risk that they cannot eliminate – i.e. the market risk

If we plot a graph for the CAPM, we will get a security market line (SML)

- The SML describes graphically the relationship between expected return vs. beta
- The SML can be plotted by drawing a line through risk-free investment (β=0) and the market portfolio (β=1)

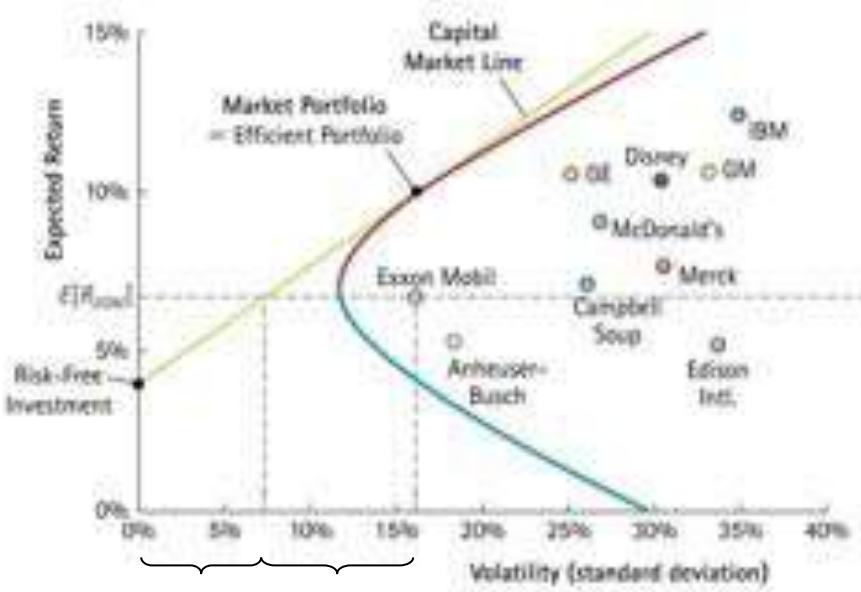


- The SML says an asset/portfolio's expected excess return (excess return = expected return – risk-free rate) is proportional to its systematic risk
- So, if the β of a security is 1, investors can expect to fetch some 12%, whereas investors in assets with a β of 1.5 can expect a return of some 15%

It is useful to compare the SML with the CML

- Both graphs have risk/return on the vertical axis, but the CML shows total risk whereas the SML shows only the risk of individual assets
- Under the CAPM assumptions, the market portfolio is the efficient portfolio
- If we plot individual securities according to their expected return and β, the CAPM implies that they should all fall along the SML

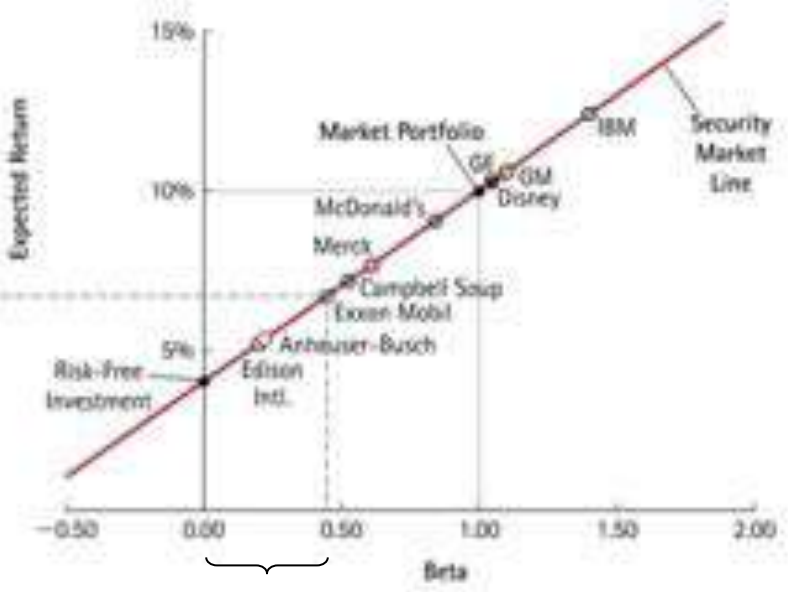
Capital Market Line



Risk of Exxon Mobil due to market risk

Risk of Exxon Mobil due to diversifiable risk

Security Market Line

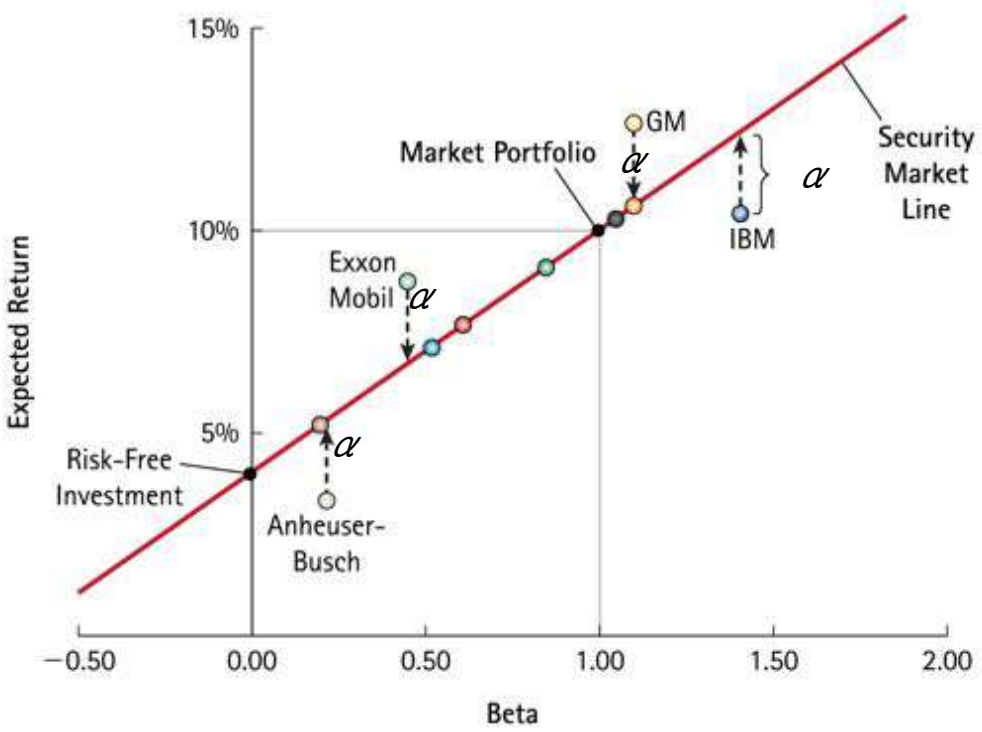


Beta of Exxon Mobil

Source: Berk and DeMarzo (2007)

In addition to β, there is also alpha (α) which represents the deviations from the SML

- α is the difference between the fair and actual expected rate of return on a share
- It is therefore the abnormal rate of return on a security in excess of what would be predicted by CAPM



- Securities that are fairly priced will lie on the SML
- Therefore, for those securities that are not on the SML, it presents the opportunity to make extra return
- Investors will buy GM and Exxon Mobil, then pushing share prices up and lowering the return
- They sell Anheuser-Bush and IBM, leading to a drop in share prices and an increase in return

Source: Berk and DeMarzo (2007)

Exercise

Problems	Descriptions
Problem 1a	• What is the expected return on a security with a beta of 0.2 if the risk-free rate is 3.3% and the market rate of return is 10.1%?
Problem 1b	• What if the expected market rate to increase to 12.3%?

Topic 3: CAPM

Portfolio beta and drivers for beta

Just as we can calculate the β of a security, we can derive the β of a portfolio

- The β of a portfolio is the weighted average of the ß of all the individual securities

Example

The share of company A has a β of 1.5 and that of Firm Z has a β of 0.7. The risk-free rate is 3% and the market risk premium is 8%. If you consider a portfolio formed by investing equally in these 2 securities, what is the expected return and the β of the portfolio?

Let us first calculate the βof the portfolio

$$\beta_P = w_A \times \beta_A + w_Z \times \beta_Z$$
$$= 0.5 \times 1.5 + 0.5 \times 0.7$$
$$= 1.1$$

We can then choose one of the following 2 ways to calculate the expected return

(1) $E(r_p) = w_A \times E(r_A) + w_Z \times E(r_Z)$

$E(r_A) = 3\% + 1.5 \times 8\% = 15\%$

$E(r_Z) = 3\% + 0.7 \times 8\% = 8.6\%$

$E(r_P) = 0.5 \times 15\% + 0.5 \times 8.6\% = 11.8\%$

(2) $E(r_P) = r_f + \beta_P \times (r_m - r_f)$

$= 3\% + 1.1 \times 0.8$

$= 11.8\%$

β of a company depends on a company's 1) business risk (which includes cyclicality of sales)…

- The revenue of some firms are quite cyclical
- They do well in the expansion phase of the business cycle and do poorly in the contraction phase

- The greater the effect of the state of the economy on a business sector, the higher its β
  - Cyclical firms include those in the high-tech industries (e.g. salesforce.com, β=2.33), automotive industry (e.g. Ford Motor, β=1.83), and computer and data processing (e.g. Bull, β=1.64)
  - Stable firms include those in the utility industries (e.g. Scottish and Southern Energy, β =0.61) and food retailing (e.g. Carrefour, β=0.64))
  - Counter-cyclical firms include those in the gold-mining industries (e.g. Barrick Gold, β=0.40)

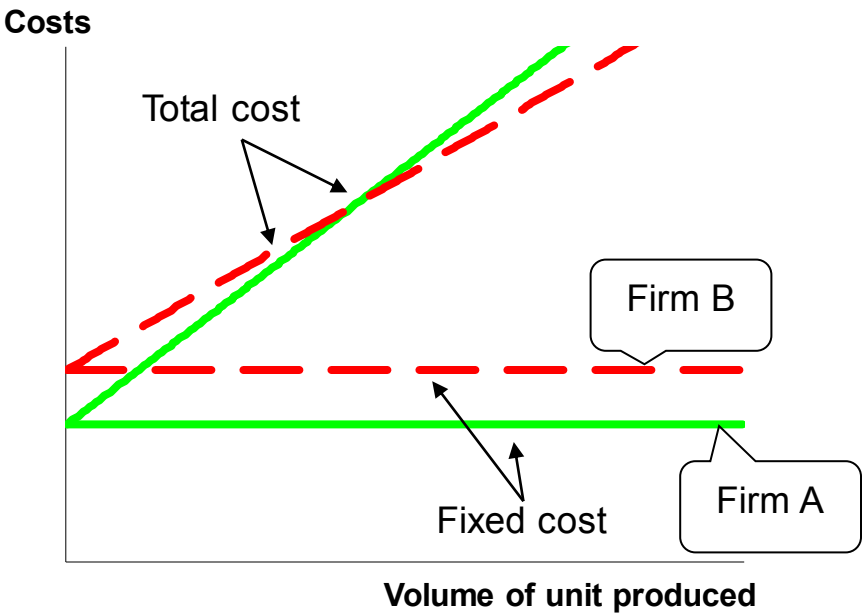
- Because β refers to the sensitivity of a firm's returns to the markets, it is not surprising that highly cyclical stocks have high β

…and operating leverage…

- Companies that have a high ratio of fixed costs (e.g. cement makers) have a high β, while those with a low ratio of fixed costs (e.g. mass-market service retailers) have a low β

Example

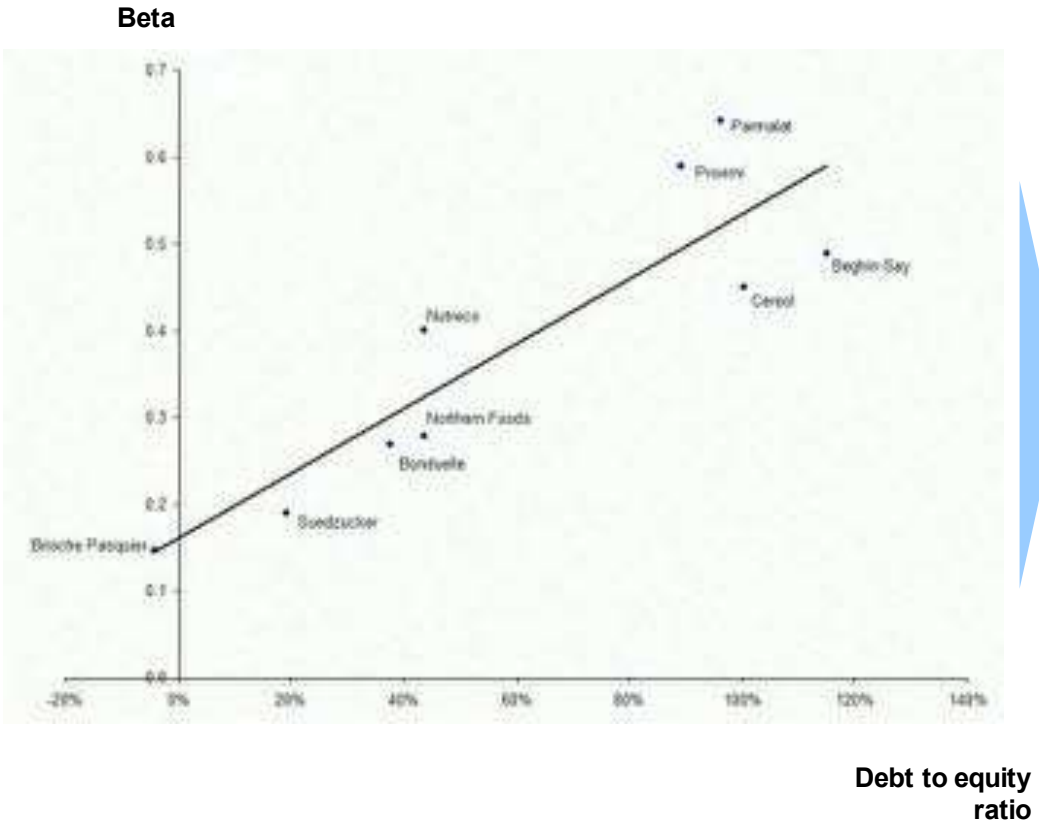
	Firm A (Green)	Firm B (Red broken)
Fixed cost (per year)	£5,000	£7,000
Variable cost (per unit)	£80	£60
Average price (per unit)	£100	£100



- Firm B has a higher operating leverage because it has higher fixed costs
- If business is good, then B will have a higher profit than A because B has a lower total cost. But if business is bad, then B's profit will be lower due to high fixed costs
- So, firm B is more sensitive to economic up- and downturns and thus have a higher β

…as well as 2) financial risk due to leverage

- The greater the company's debt, the greater its financing costs
- Financing costs are fixed costs which increase a company's break-even point and hence its earning volatility
- The heavier a company's debt, the higher is the β for its shares



- Because firms with debt must make interest payments regardless of their sales, financial leverage increases the sensitivity of their expected returns to that of the market

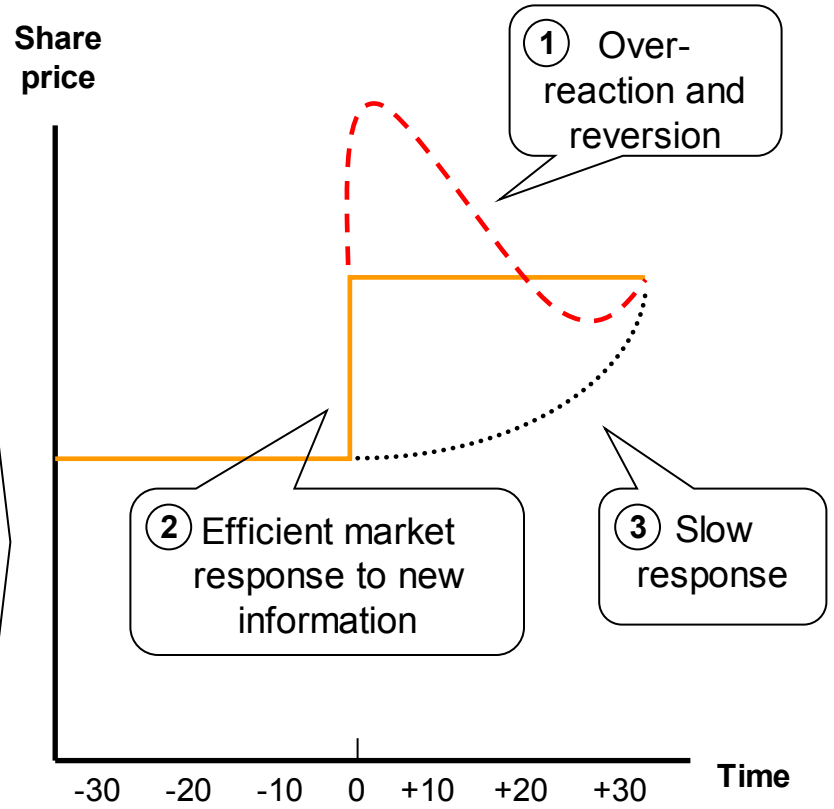
Source: Vernimmen et al. (2005)

Topic 4: Efficient market hypothesis

Efficient market hypothesis

Share price and equity financing depend on the efficiency of the capital markets

- An efficient capital market is one which the prices of financial securities at any time **rapidly** reflect **all available relevant** information
- Consider the reaction of share price to new information



1. Over-reaction: The price over-adjusts to the new information. There is a bubble in the price sequence
2. Efficient market response: The price instantaneously adjusts to and fully reflects new information; there is no tendency for subsequent increases and decreases
3. Slow response: The price adjusts slowly to the new information

- In an efficient market, information is reflected in prices immediately, therefore investors should only expect to obtain a normal rate of return. Investors should therefore not waste resources trying to find bargain securities
- Firms should expect to receive fair value for securities they sell
- In such a market, competition between financial investors is so fierce that prices adjust to new information almost instantaneously

But since the market does not exhibit the fully efficient market reaction, it is possible to categorise the market into different forms

- The **efficient market hypothesis** (EMH) asserts that well-organised capital markets are efficient markets. While insufficiencies may exist, they are relatively small and uncommon
- There are 3 forms:

Forms	Descriptions
Weak-form EMH	• In the weak-form efficient market, it is impossible to predict future returns • Existing prices already reflect all the information that can be gleaned from studying past prices and trading volumes, interest rates and returns
Semi-strong form EMH	• A semi-strong efficient market reflects all publicly available information • Semi-strong efficiency is superior to weak form efficiency because it requires that current prices include historical information **and** publicly available information
Strong-form EMH	• In a strongly efficient financial market, investors with privileged or insider information or with a monopoly on certain information are unable to influence securities prices • This holds true only when financial market regulators have the power to prohibit and punish the use of insider information

Topic 5: Weighted average cost of capital

Weighted average cost of capital

Cost of capital represents the discount rate of cash flow, expected return and opportunity cost of a firm or a project

Aspects	Description
1 Discount rate	• Cost of capital is the discount rate for discounting cash flows • Discount rate of a project should be the expected return on a financial asset of comparable risk
2 Expected rate of return	• Cost of capital is also the minimum expected rate of return that the suppliers of capital (both the bondholders and shareholders) require as a compensation for making the financial contribution • It is also minimum expected rate of return an investment must offer to be attractive – what the firm must earn on its capital investment in a project just to break even
3 Opportunity cost	• Cost of capital can also be interpreted as the opportunity cost associated with a firm's capital investment

The cost of capital is the minimum rate of return on a company's investments that can satisfy both shareholders (cost of equity) and debtholders (cost of debt). The cost of capital is thus the company's total cost of financing

- Weighted average cost of capital (WACC) is based on the average of the return required by shareholders (r_E) and the after-tax return demanded by creditors (r_D), weighted by the respective portions of equity and debt in a firm

$$WACC = w_E r_E + w_D r_D (1 - t)$$

w stands for weight. So, w_E is the proportion of equity on a firm's capital structure (E / D+E) while w_D is the proportion of debt or D / D+E

On the other hand, T stands for tax rate

There are 2 ways to determine the weight. The first is to examine the current capital structure …

Methods

1 **Current capital structure**

Description

Equity

- The equity portion of WACC is determined by number of shares outstanding, multiplied by the share price, which is called **market capitalisation**
- Since this method uses share price, the weight reflects the market value of the firm's equity
- A less desirable alternative is to use book value of equity. But usually, this is restricted to the situation where market information is not readily available. This is because book value of liabilities can be very misleading since it may have changed over time and can significantly differ from market value of equity
- Market value of equity is more appropriate because this is the value that shareholders base their required rate of return on
- Nevertheless, comparison between the two alternatives, however, may yield some insights

There are 2 ways to determine the weight. The first is to examine the current capital structure …(cont'd)

Methods	Description
1 Current capital structure	**Debt** • The debt portion is determined by the market price of a single bond multiplied by the number of bonds outstanding • The use of the market price is important because the weight should reflect the market value of the firm's debt • If there are multiple bond issues (as there normally would be), it is necessary to repeat this calculation for each type of debt • Similar to equity, it is more appropriate to use market value of debt

… and the second is to aim for the target capital structure

Methods	Description
2 Target capital structure	• A firm may want to move to a different capital structure for various reasons such as changing business model (e.g. becoming more focused or diversified) • Firm should always attempt to determine what its optimal or best mix of financing should be (but is it that easy?) • The trend in the changes of the competitors capital structure of competitors may be useful in determining the target capital structure • It is also possible to use the average of the capital structure of comparable companies to determine the target capital structure

There are also different ways to determine the cost of debt...

Aspects	Description
1 Cost of debt	• The cost of debt is what the firm must pay on new borrowing • This is the rate of return that debtholders demand • The cost of debt can be observed both: – Directly – if the firm already has bonds outstanding, then the yield to maturity (YTM) on those bonds is the market rate on the firm's debt – Indirectly – if we know that the firm's bonds are rated, say, AA, then we can simply find the YTM on newly issued AA-rated bonds • YTM is used instead of coupon rates because companies are interested to know what the market value of the debt is, since this is the current required rate of return by debtholders* • It is, however, difficult (if not impossible) to determine the cost of bank borrowing because they do not have a market price. In this case, the interest rate on the loan is used instead • Given that short-term debt also carries cost of debt, both short- and long-term debt should be considered

** Think of it this way – there are 3 components in a bond that determines the rate of return on the bond (YTM): coupons (i.e. interest), maturity (i.e. the life) and the face value of the bond. Therefore, if one focuses on the coupon to determine the rate of return, then one is missing out the other 2 potentially influential components.*

… and cost of equity

Aspects	Description
2 Cost of equity	• Cost of equity can be estimated by investors' required rate of return on a firm's share • Dividend growth model can be used for calculating the cost of equity but it has limitations • CAPM represents another alternative to estimate the cost of equity. In this case, the cost of equity depends on three things: - The risk-free rate: r_f - The market risk premium: r_m-r_f - The systematic risk of the asset relative to average or β: - Expected return on a share is therefore equal to: $$r_f + \beta(r_m - r_f)$$ • It is worth stressing again that in the CAPM method, the cost of equity required by investors depends upon just one factor: systematic risk since non-systematic risks are not remunerated

Exercise

Aspects	Description
Problem	• Roswell Technology has the following balance sheet that has been recently updated. Calculate the company's cost of capital. The debt has just been refinanced at the YTM of 6% (short-term) and 8% (long-term). The expected rate of return on the company's shares is 15%. There are 7.46 million shares outstanding, and the shares are trading at $46. The tax rate is 35%.

(Figures in £ 000s)

Cash and marketable securities	1,500	Short-term debt	80,000
Account receivables	124,400	Accounts payable	62,000
Inventories	125,000	Current liabilities	142,000
Current assets	250,900	Long-term debt	208,600
Fixed assets	302,000	Deferred taxes	45,000
Other assets	89,000	Shareholders' equity	246,300
Total	641,900	Total	641,900

Topic 6: Beta levering

Unlevering and relevering beta

The β of a security reflects the company's business and financial risks. However, there are circumstances in which we need to estimate a different β

- In the previous session, we learned that a company's β incorporates both business and financial risks*
- But what happens if the mix of debt and equity (capital structure) changes?
- The answer is that we will have to 'take out' the financial risk portion of β and calculate the new β with the new mix of debt and equity

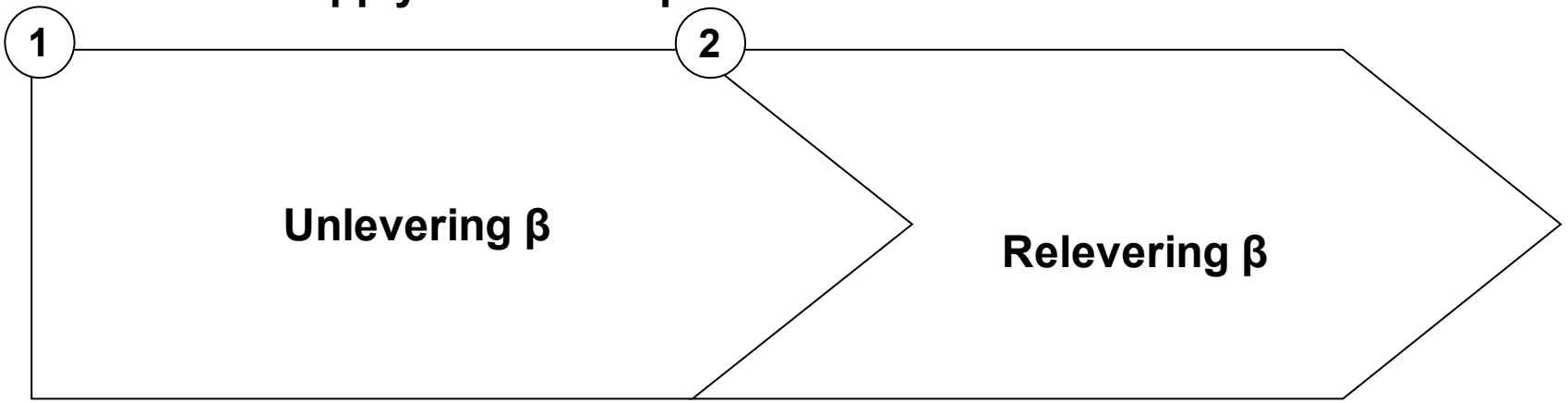
- The β that has removed the effect of financial leverage is called unlevered** β (also called asset β***)
- The unlevered β therefore measures the market risk of the firm without leverage, which is equivalent to the β of the firm's assets
- In other words, unlevered β measures the market risk of the firm's business activities, ignoring any additional risk due to leverage

* This β that includes both business and financial risks is also called equity β

** Unlevered refers to the case where the firm is all-equity, i.e. no debt

*** This is called asset β because this is the β that would be expected when the company financed only with equity capital (i.e. no leverage). So, all the systematic risk of the company is purely due to its assets and operation performance and not to any financial risk from leverage

There are 2 steps to unlever and relever β so as to identify the industry-specific business risk and apply the new capital structure



- Unlevering the β of a firm to figure out what the β is if there is no financial risk due to the existence of debt
- In short, you are computing the β of a firm with only business risk
- Use the following formula to calculate the asset beta (i.e. the beta with only business risk):

$$\beta_{asset} = \frac{\beta_{equity}}{\left[1 + \frac{D}{E}(1 - T)\right]}$$

- Apply β_A (the β with no financial risks) with the new capital structure to calculate the β that fits the target industry with the desired capital structure
- Use the following formula to calculate levered β (note that it is the exact same formula)

$$\beta_{equity} = \beta_{asset}\left[1 + \frac{D}{E}(1 - T)\right]$$

Example

Aspects	Descriptions
Problem	• Suppose Acura Technology is considering investing in a new business in the biotech industry to start a new division. Biogene is a major player in the biotech industry and has an equity beta of 0.90. It has $95m of equity and $5m of debt. Acura Technology intends to use an equity-to-value ratio of 60%. What β should the company use to estimate the cost of capital in the new business, assuming that there is no tax?
Solution	• We will have to 1) unlever Biogene's β and then 2) re-lever the asset β with capital structure of the new business of Acura Technology 1. Using the formula to calculate the asset or unlevered β $\beta_{Asset} = \frac{\beta_{Levered}}{\left(1+\frac{D}{E}\right)} = \frac{0.90}{\left(1+\frac{0.05}{0.95}\right)}$ $= 0.855$ 2. Once the asset β of Biogene is determined, we can re-lever $\beta_{Levered} = \beta_{Asset}\left(1+\frac{D}{E}\right) = 0.855 \times \left(1+\frac{0.4}{0.6}\right)$ $= 1.425$ Therefore, the β for the new division should be 1.425

Exercise

Aspects	Descriptions
Problem	• Suppose Acura Technology is considering investing in a new business in the environmental industry. Cronmental is a prominent player in this industry. It has an equity beta of 1.20. It has €50m of equity and €50m of debt. Acura Technology intends to use an equity-to-value ratio of 70%. What β should it use to estimate the cost of capital in the new business if tax rate is 30%?

Topic 7: Company valuation

Introduction to valuation

Firm valuation plays a fundamental role in mergers and acquisitions activities as well as determination of share price. The aim of valuation is to determine what a firm is worth

- The purpose of valuation is to estimate the value of a company
- Value is determined based on historical and future financial parameters
- But many people have held misconceptions of firm valuation including:

Misconception	Description
Misconception 1	• Since valuation models are quantitative, valuation is objective • Correction: determination of firm value is a highly subjective exercise that calls for judgment
Misconception 2	• A well-researched and well-executed valuation is timeless • Correction: Value of the firm changes and varies when business drivers and factors (both internal and external) as well as the environmental conditions change
Misconception 3	• A good valuation provides a precise estimate of value • Correction: Since value estimation is subjective, it serves more as a basis for discussions
Misconception 4	• The more quantitative a model, the better the valuation • Correction: There are many aspects of business that are hard to quantify such as consumer behaviour

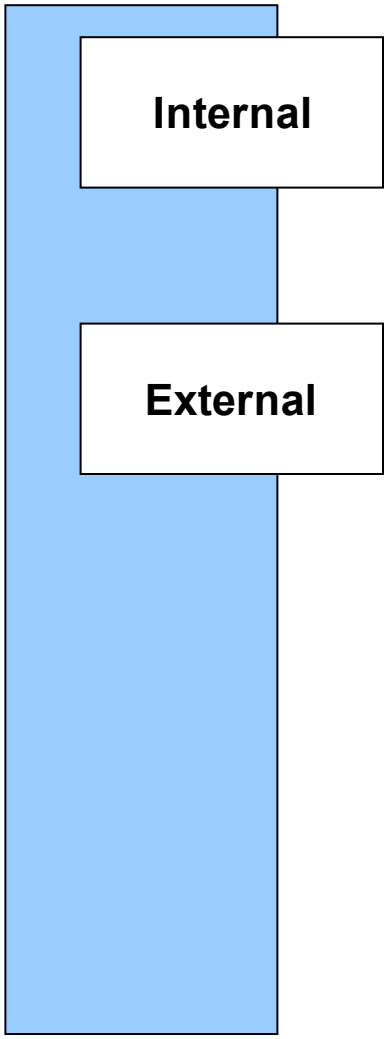
Firm valuation plays a fundamental role in mergers and acquisitions activities as well as determination of share price. The aim of valuation is to determine what a firm is worth (cont'd)

- The purpose of valuation is to estimate the value of a company
- Value is determined based on historical and future financial parameters
- But many people have held misconceptions of firm valuation including:

Misconception	Description
Misconception 5	• To make money on valuation, you have to assume that markets are inefficient • Correction: It is necessary for the markets to be somewhat efficient. Otherwise, it will be difficult to obtain a good price
Misconception 6	• It is the final value that matters, not the process of valuation itself • Correction: Valuation is an exercise that initiates discussions and negotiations

Companies can be valued from two perspectives of a firm

- Analysts value companies from 2 aspects



Aspects	Description
Internal	• The internal methods estimate the value of all the future potential cash flows generated by the company • Therefore, it relies on methods based on the concept of DCF
External	• Companies can also be valued through benchmarking competitors • In effect, the external methods capture the market view of these companies, through which the value of a firm can be determined • Methods relying on external views are called multiple valuation methods because they involve the use of multiples

There exists a number of methods to estimate what a company should worth. The value of a company can be estimated by combining the range of values resulted from the different methods

Valuation techniques	Description
DCF	• Discounting all future free cash flows (internal)
Company comparables	• Examining the multiples of competitors and other companies that engage in activities similar to those of the firm to be valued (external)
M&A comparables	• Examining the multiples of previous M&A transactions in the sector of the firms to be valued (external)
Sum of the parts	• Break up the company and value the parts (internal)*

- Using the results from these methods together, you can estimate the value of a company
- However, biases and errors will always exist because
  - Different people may want to justify their points of view (e.g. bidder and target in a M&A usually have different views
  - Comparables are open for subjective views and interpretations
  - Cash flows are estimated based on assumptions

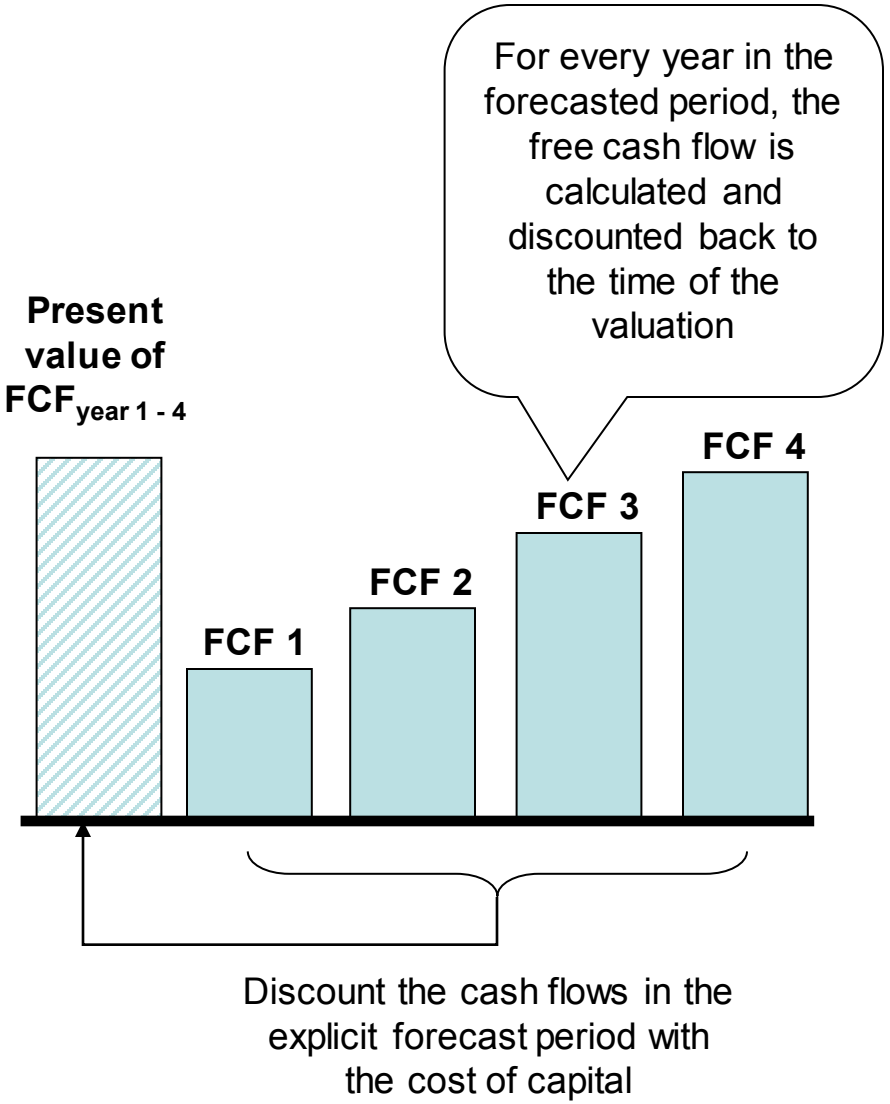
** This method is beyond the scope of this module and is therefore not covered here*

Topic 7: Company valuation

DCF method to valuation (internal)

The DCF methods to valuation are based on the calculation of the present value of all the free cash flows of a company, which is very similar to valuing a project that lasts forever

- A firm is like a project that generates cash flows without end
- Hence, by estimating the value of the (free cash flows) FCF in the first years, it is possible to calculate the value of the firm accumulated in those years



- For firm valuation, we project specific cash flows over a certain number of years
- This is called the **explicit forecast period**
- The length of this period varies depending on the sector
- This period can be as short as 5-7 years for consumer product firms and as long as 20-30 years for utilities

However, contrary to a project that will eventually come to an end, it is expected that a firm will stay as an on-going concern forever

- Unless there exists information on **all** the future cash flows, it will be impossible to calculate the PV of all the FCF of a firm
- Even if it is possible to forecast such (theoretically possible) information, it is impractical if not unrealistic
- Therefore, a **terminal value** (TV) has to be established. It represents all the FCF beyond the explicit forecast period
- There are 2 ways to estimate TV

Methods	Description
1 **Perpetual growth model**	• This is the dividend growth model (also called the Gordon-Shapiro model) • This model assumes the growth rate of the firm to become stable and remains the same forever: $TV_t = \frac{\text{Normalised FCF}}{r - g}$ where TV_t is the terminal value at time t, normalised FCF is cash flow at time t+1, r is the cost of capital and g is the constant growth rate • In short, this is the value of the firm at the end of the explicit forecast period
2 **Exit or terminal multiple**	• This method assumes that the firm is sold right after the explicit forecast period • So the aim is to determine how much the business is worth based on a certain operating performance measure • For example, it can be “10x 2015 EBITDA”, which means beyond explicit forecast period, all the subsequent EBITDAs add together equal 10 times the size of the EBITDA in 2015

Even though the exit (or terminal) multiple may appear to be arbitrarily determined, it is actually related to dividend growth model

- Recall that the perpetual growth model represents cash flow growing at a specific rate forever

$$\frac{CF_1}{r-g}$$

- This implies that the value of a firm is equal to a constant

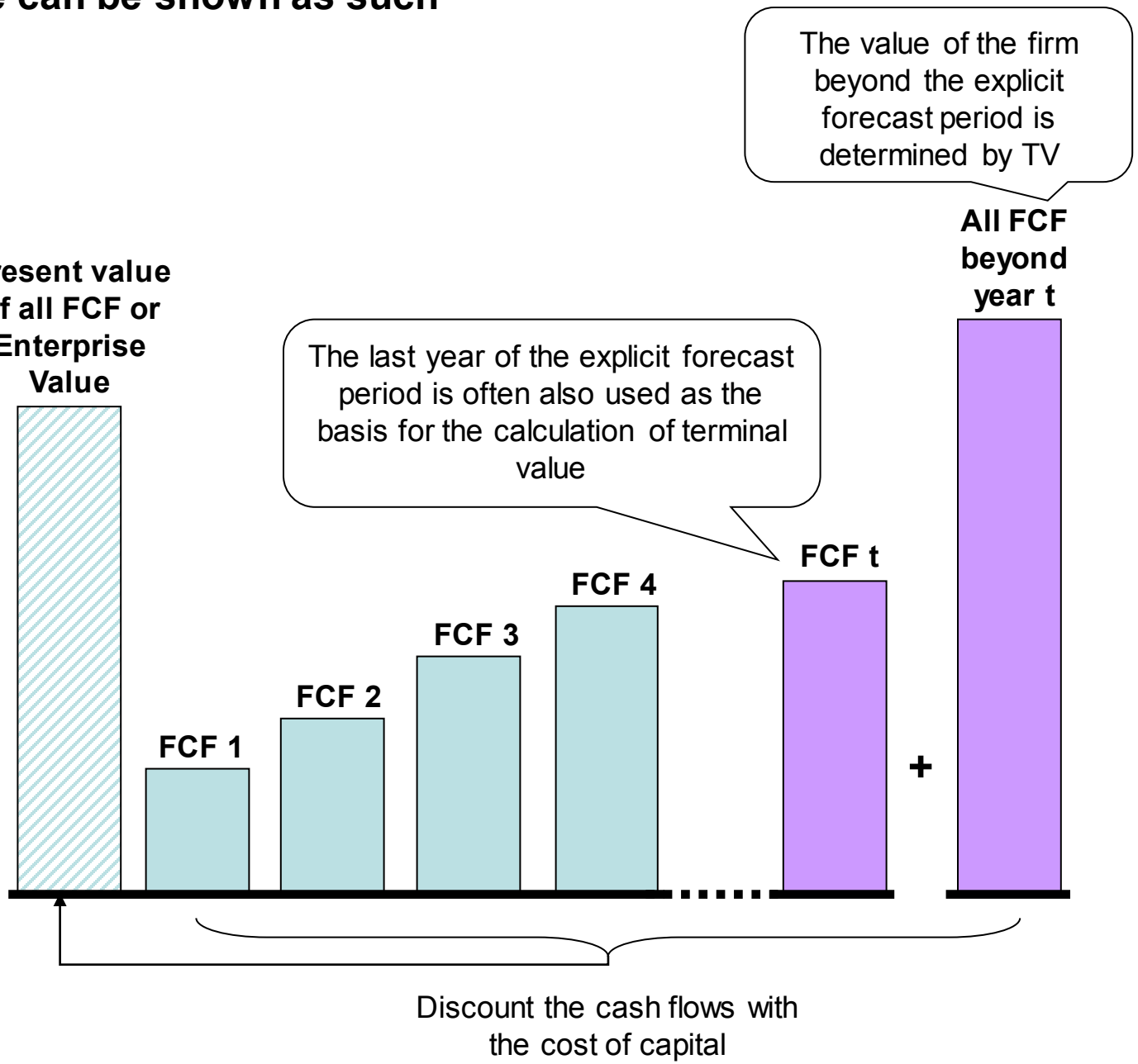
$$\frac{1}{r-g} \text{ times}$$

the cash flow today

- This is therefore the "multiple"
- The advantages of this method are
  - There are few computations
  - It does not require an estimation of r or g

Graphically, the calculation of the present value of all the future cash flows with the terminal value can be shown as such

- So, the DCF methods to firm valuation is about calculating a) all the foreseeable FCF in the explicit forecast period and b) the value of firm beyond this period, as represented by the TV



Before moving on, it is necessary to explain the concepts of normalised FCF…

Concepts	Description
Normalised FCF	• Both approaches to TV assume that the operating performance in the last year of the explicit forecast period to be normalised. In method ①, the FCF beyond the explicit forecast period is assumed to be normalised whereas in method ②, EBITDA is used in place of FCF • Normalised refers to the cash flow or EBITDA in the last year of the explicit forecast period is no longer growing significantly and starts to grow "normally" • The rationale behind this is that no above-average profit can be sustained forever as competitors enter the market and any competitive advantages will be eroded • As the company enters the period when growth starts to stabilise, we will assume that all the elements contributing to the FCF such as EBIT, tax rate, capex, NWC, etc. will also stabilise

… and enterprise value

Concepts	Description
Enterprise value	• The value of a firm is often called enterprise value (EV) • Since the EV of a firm represents the value of the underlying business, it should equal to the total value of debt and equity

Enterprise value (also called firm value)	Value of net debt
	Equity value

- Equity value is often represented by **market capitalisation**, which is calculated by multiplying number of shares outstanding with the current share price
- **Net debt**, on the other hand, refers to long-term debt minus cash
- Only long-term debt is taken into account because short-term debt as well as the current portion of long-term debt are considered as current liabilities*
- In other words, the value of a firm or EV can be calculated by **market capitalisation plus net debt**

* Which is taken into account by working capital that, in turn, forms part of the asset side of the balance sheet or in this case, the EV

The DCF method consists of 4 steps, which include …

Steps	Description
i Preparing Forecast	• Since you are forecasting the future cash flows of a company, it is necessary to prepare a P&L that include forecasted revenues, costs and expenses, capex, working capital requirements, etc.
ii Calculating FCF	• With all the forecasts in place, you can determine the FCF based on the cash flows from operation, capex, change in working capital
iii Calculating discount rate and TV	• To discount the cash flows, you have to establish the discount rate using balance sheet and market information • Determining the TV, on the other hand, requires you to figure out the stabilised growth rate or a EBIT/EBITDA multiple
iv Discounting the cash flows	• Discount the FCF with the computed discount rate

- A great deal of assumptions will have to be made for the forecasts and estimating the FCF, the discount rate and the TV
- The final present value of all the FCF would be the value of the firm or its enterprise value
- In the following example, we focus on the most commonly used DCF method call the WACC method*
- Other methods are discussed later

** This is called the WACC method because it uses WACC as the discount rate*

… Preparing the forecast P&L …

Example

- You are trying to estimate the value of Tranquillity, a tobacco company
- The first step is to create a P&L forecast for this firm

PROFIT AND LOSS
(€ millions)

		2010E	2011E	2012F	2013F	2014F	2015F	2016F	2017F	2018F	2019F
	Sales	4,180	4,288	4,388	4,475	4,569	4,658	4,750	4,844	4,940	5,040
(-)	Costs of goods sold	(2,592)	(2,659)	(2,721)	(2,775)	(2,833)	(2,888)	(2,945)	(3,003)	(3,063)	(3,125)
	Gross profit	**1,588**	**1,629**	**1,667**	**1,701**	**1,736**	**1,770**	**1,805**	**1,841**	**1,877**	**1,915**
(-)	Operating expenses	(491)	(496)	(449)	(420)	(420)	(438)	(447)	(456)	(465)	(475)
	EBIT	**1,097**	**1,133**	**1,218**	**1,281**	**1,316**	**1,332**	**1,358**	**1,384**	**1,412**	**1,440**
(-)	Interest	(112)	(123)	(137)	(149)	(159)	(154)	(157)	(160)	(164)	(167)
	EBT	**985**	**1,010**	**1,081**	**1,132**	**1,157**	**1,177**	**1,200**	**1,224**	**1,249**	**1,273**
(-)	Tax	(324)	(331)	(354)	(370)	(378)	(382)	(390)	(397)	(405)	(413)
	Net income	**661**	**679**	**727**	**762**	**779**	**795**	**811**	**827**	**843**	**860**
	EBIT	1,097	1,133	1,218	1,281	1,316	1,332	1,358	1,384	1,412	1,440
(+)	D&A	191	191	192	196	201	204	208	213	217	221
	EBITDA	**1,288**	**1,324**	**1,410**	**1,477**	**1,517**	**1,536**	**1,566**	**1,597**	**1,629**	**1,662**

ii … Calculating the free cash flow …

- From the P&L, FCF can then be computed

ii

FREE CASH FLOW
(€ millions)

		2010E	2011E	2012F	2013F	2014F	2015F	2016F	2017F	2018F	2019F
	EBIT	1,097	1,133	1,218	1,281	1,316	1,332	1,358	1,384	1,412	1,440
(-)	Tax	(324)	(331)	(354)	(370)	(378)	(382)	(390)	(397)	(405)	(413)
	Unlevered net income (NOPAT) (A)	773	802	864	911	938	950	968	987	1,007	1,027
	D&A	191	191	192	196	201	204	208	213	217	221
	(-) Capex (B)	(179)	(174)	(174)	(174)	(174)	(177)	(181)	(184)	(188)	(192)
	(-) Δ working capital (A)	(84)	(73)	-	-	-	-	-	-	-	-
	Unlevered FCF	**701**	**747**	**882**	**933**	**965**	**976**	**996**	**1,015**	**1,035**	**1,056**

ii … Calculating the free cash flow (cont'd) …

- Two points in the previous FCF calculations require further explanations

Concepts	Description
A **Unlevered**	• They are called **unlevered** because they are before interests are paid. In other words, these are monies that go to **both** creditors and shareholders. • This contrasts to **levered** cash flow, which refers to the amount of cash available to shareholders after interest payments are made • It must be noted that they are not to be confused with the similar use of terms for unlevered firm (which means debt-free) and levered firm (which means the company is financed by both debt and equity)
B **Capex and Δworking capital**	• Both Capex and Δ working capital are assumptions • In the example, it is assumed that capex will grow by 2% given the nature of the industry • On the other hand, it is assumed that in 2012, there will be no need to inject further working capital • It is also necessary to note that since a company will exist forever, there will be no working capital recovery

iii … Calculating the WACC and TV (cont'd) …

Concepts

WACC

Description

- Estimating the WACC is one of the most sensitive aspects of the DCF approach. **However, it must be noted that WACC is used as cost of capital only when the WACC valuation method is used. For these other methods, the cost of capital will have to be adjusted accordingly**
- The WACC represents the minimum rate of return required by the company's sources of funding, i.e. shareholders and lenders

$$WACC = \frac{E}{D+E} r_E + \frac{D}{D+E} r_D (1-t)$$

- It must be noted that all variables in the WACC calculation are related to the whole firm
- Therefore, the after-tax WACC only gives the right discount rate only for new projects that are the same as the firm's "average" projects*

** So if you are buying a company and wants to know its value by discounting all FCF with WACC, you are assuming that this company you are considering has the same level of risk as company the company that you would like to it to merge it*

iii … Calculating the WACC and TV (cont'd) …

Concepts	Description
WACC	• WACC will have to be modified if the overall debt ratio changes • Since CAPM is often used to estimate the return on equity, the β of other companies may have to be used. Consequently, it may be necessary to unlever and re-lever the β • Market information or company disclosed information can be used to determine the level of debt and the return on debt • Alternatively, many analysts in M&A simply use the current company WACC (and assume the same capital structure and/or same risk of the target entity)

iii … Calculating the WACC and TV (cont'd) …

Example

- A firm has a marginal tax rate of 35%
- Cost of equity is 14.6% and the pre-tax cost of debt is 8%
- What is the WACC?

Marginal tax rate	35.0%
Cost of equity	14.6%
Pre-tax cost of debt	8.0%

✓ **MARKET VALUE**

Assets		Liabilities & equity	
Assets	250	Debt	100
		Equity	150
Total	250	Total	250

✗ **BOOK VALUE**

Assets		Liabilities &	
Assets	200	Debt	100
		Equity	100
Total	200	Total	200

- When you are estimating future cash flows, you are not interested in past investments but rather the current values and future expectations
- Therefore, it is more appropriate to use market value instead of book value of balance sheet when trying to establish the debt ratio and the equity ratio
- Therefore, debt ratio = 0.4 (100/250) and equity ratio = 0.6 (150/250) and WACC equals 10.84%

iii … Calculating the WACC and TV (cont'd) …

Concepts

Terminal value

Description

- On the other hand, to determine the TV, the two possible approaches are:
- **Perpetual growth model**
  - This model allows us to estimate the TV based on fundamentals using the DCF approach
  - Specifically, we can use the Gordon-Shapiro model to estimate the TV with the FCF in the last year of the explicit forecast period
  - The growth rate, g, will have to be estimated
- **Exit or terminal multiple**
  - While forecasting cash flows is useful in capturing those specific aspects of a company that distinguish the firm from its competitors in the short-run, in the long-run firms in the same industry typically have similar expected growth rates, profitability and risk
  - As a consequence, it is relatively homogenous across firms
  - Of the different ways to determine the TV, EBITDA is most often used because it accounts for the firm's operating efficiency and is not affected by differences between firm

(iv) … and discounting the cash flows in the explicit forecast period …

- Discount all the FCF in the explicit forecast period to obtain the PV of the forecast cash flows

iv

Discounted projected cash flow
(€ millions)

	2010E	2011E	2012F	2013F	2014F	2015F	2016F	2017F	2018F	2019F
Free cash flow	701	747	882	933	965	976	996	1,015	1,035	1,056
Discount factor	0.94	0.88	0.83	0.78	0.73	0.69	0.64	0.60	0.57	0.53
Discounted cash flow	658	658	730	725	704	669	641	613	587	563

PV of discounted cash flow in the explicit forecast period @ 6.50% **6,549**

(iv) … and with the PV of TV also calculated, it is possible to determine the enterprise value, that is, the value of the firm

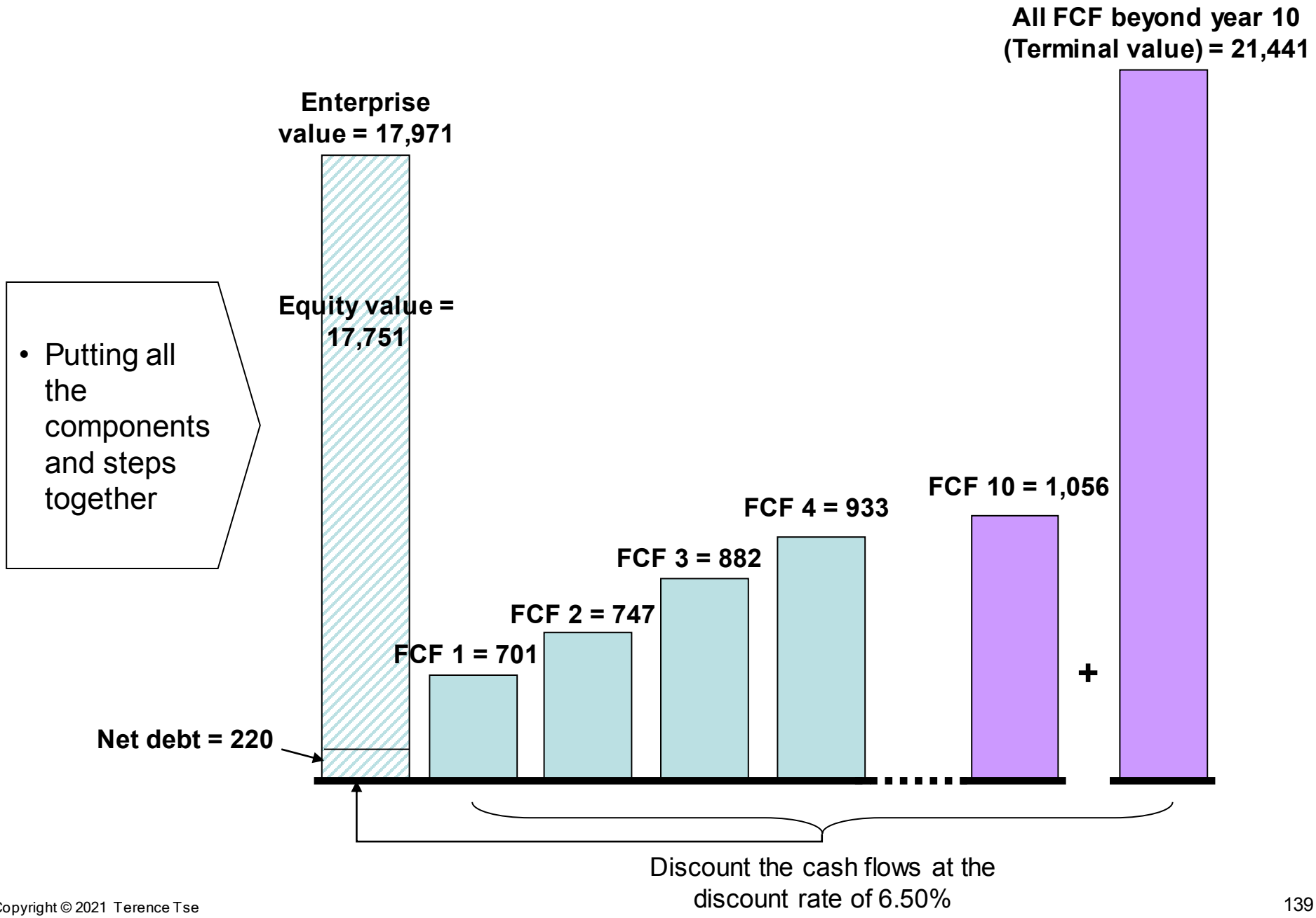
- In this case, the dividend growth model is used to calculate the TV of Tranquillity

TERMINAL VALUE
(€ millions)

Date	*30-Mar-09*
WACC	*6.50%*
Terminal growth rate	*1.50%*
Normalised unlevered 2019 free cash flows	1,056
Terminal Value (@ 1.5% Perp. Growth)	21,441
PV of TV	11,422
ENTERPRISE VALUE	**17,971**
EQUITY VALUE	
Net debt	*220*
Equity Value	**17,751**

- TV is calculated by 1,056/(6.50% - 1.50%)
- Discounting the TV to attain the PV leads to 11,422
- Hence, the value of Tranquillity is estimated to be €17.9 billion according to the WACC method
- Equity value or the value of the firm that belongs to the shareholders can be calculated by EV minus net debt

The entire valuation process of this example can be presented in this graph



It is a common practice not to focus just on a single value, but rather to check a range of values

- Since the WACC and TV are the 2 major "estimates", it is necessary to see how sensitive various results are when both the WACC and the TV vary

TERMINAL FCF

		WACC				
		5.50%	6.00%	6.50%	7.00%	7.50%
Terminal growth rate	0.50%	12,428	10,777	9,425	8,302	7,357
	1.00%	13,878	11,913	10,333	9,038	7,963
	1.50%	15,690	13,303	11,422	9,909	8,669
	2.00%	18,020	15,039	12,754	10,953	9,504
	2.50%	21,126	17,272	14,418	12,230	10,505

ENTERPRISE VALUE

		WACC				
		5.50%	6.00%	6.50%	7.00%	7.50%
Terminal growth rate	0.50%	19,318	17,493	15,974	14,690	13,591
	1.00%	20,768	18,630	16,882	15,427	14,196
	1.50%	22,580	20,019	17,971	16,297	14,902
	2.00%	24,909	21,756	19,303	17,342	15,737
	2.50%	28,016	23,988	20,968	18,618	16,739

EQUITY VALUE

		WACC				
		5.50%	6.00%	6.50%	7.00%	7.50%
Terminal growth rate	0.50%	19,098	17,273	15,754	14,470	13,371
	1.00%	20,548	18,410	16,662	15,207	13,976
	1.50%	22,360	19,799	17,751	16,077	14,682
	2.00%	24,689	21,536	19,083	17,122	15,517
	2.50%	27,796	23,768	20,748	18,398	16,519

IMPLIED EBITDA EXIT MULTIPLE

		WACC				
		5.50%	6.00%	6.50%	7.00%	7.50%
Terminal growth rate	0.50%	11.6x	10.5x	9.6x	8.8x	8.2x
	1.00%	12.5x	11.2x	10.2x	9.3x	8.5x
	1.50%	13.6x	12.0x	10.8x	9.8x	9.0x
	2.00%	15.0x	13.1x	11.6x	10.4x	9.5x
	2.50%	16.9x	14.4x	12.6x	11.2x	10.1x

- Attaining a range of values is important because this allows for discussion – remember, firm valuation is a subjective exercise

Topic 7: Company valuation

Multiples valuation (external)

Valuation of a firm can also be accomplished through multiples

- A multiple is a market price per unit, which, when multiplied by the number of units, gives the value of the those units

Quantity	x	Multiple	=	Value
Pounds of fish	x	£ per pound	=	Cost of fish
Bushels of applies	X	£ per bushel	=	Value of apples
Litre of petrol	X	£ per litre	=	Price of a tank of petrol
Number of square feet	X	£ per square feet	=	Value of property

- But unlike these commodities, companies are all very different and unique
- Therefore, it is important to keep in mind that:
  - Multiples are used appropriately when comparing two assets that are similar in nature
  - Multiples are easy to use when they are stable across similar assets
  - Multiples may vary over time

A valuation multiple is the ratio of firm value to some aspect of the firm's economic activity (e.g. cash flow and EBITDA)

Quantity	x	Multiple	=	Value	Terminology
Cash flow	x	Firm value / Cash flow	=	Value of firm	"Cash flow multiples"
EBDITA	X	Firm value / EBITDA	=	Value of firm	"EBITDA multiples"
Sales	X	Firm value / Sales	=	Value of firm	"Sales multiples"
Customers	x	Firm value / customers	=	Value of firm	"Customer multiple"
Earnings	x	Price per share / Earnings	=	Share price	"Price-earnings ratio"

- The technique for applying a valuation multiple is identical to that of applying a price-per-square-foot multiple to value real estate, or a price per pound to a purchase of fish
- If you are studying a firm with a cash flow of £5 million and you believe it should be valued at a cash flow multiple of 10, you will determine that the firm is worth $50 million

When computing market multiples, there are a number of commonly used operating metrics

Enterprise value divided by

- Sales
- Gross profit
- EBITDA
- EBIT
- Total assets
- Net fixed assets

Market capitalisation divided by

- Net income
- Dividends
- Net cash flow
- Earnings before tax
- Assets less liabilities

Share price divided by

- Earnings per share
- Dividends per share
- Cash flow per share
- Book value per share

- It is important to note that, for instance, EBIT and EBITDA must be calculated with EV and net income can only work with market capitalisation (this is because debt is considered in EV but this is not the case in market capitalisation)
- In some sectors, there are other industry-specific multiples such as:
  - Cable TV: EV / subscribers
  - Retailers: EV / square foot
  - HMOs: EV / covered members
  - Bottlers: EV / cases
  - Technology: EV / patents
  - Technology: EV / scientists

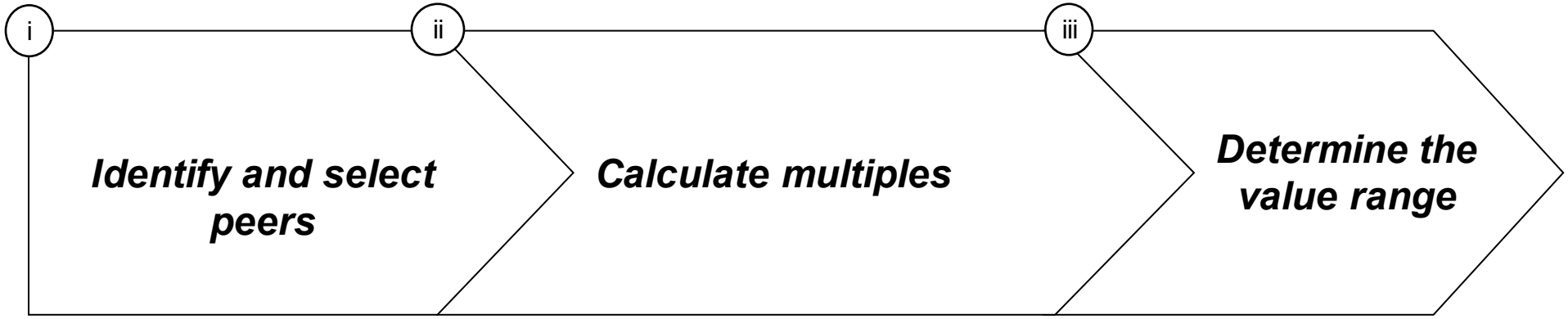
Multiples valuation, which include both industry comparables and M&A comparables, is a method in its own right, and not just a supplement to DCF analysis

- Contrast to DCF analysis, which looks at the internal value of a company, multiples valuation examines the value through the eyes of the market
- There are 3 steps to the multiples valuation approach

Steps	Description
i **Identify and select peers**	• Select comparable companies by considering their size, market position, operational effectiveness (best practice), etc. • Choose benchmarks by considering those that capture the effect of future earnings and potential for growth
ii **Calculate multiples**	• Compute the EV of the benchmark firms identified by adding market capitalisation and net debt together • Compute the respective multiples
iii **Determine value range**	• Calculate the valuation ranges by applying the above multiples to the operating and performance statistics (e.g. EBIT or EBITDA) of the firm you are valuing

- There are 2 main types of multiple valuation:
  1. Industry (also called trading) comparables (also called multiples)
  2. Transaction (also called M&A) comparables/ multiples

1 To determine the multiples for *industry* comparables (also called trading comparables), it needs to first identify the competitors and/or peer companies and then gather the information required for multiples



- If you intend to value the firm BMW, you can calculate the EV of this firm by discounting all its future cash flows
- But for multiples valuation, it is necessary to look at other car manufacturers and select those companies that are most similar to BMW

- Calculate the different values and ratios for the selected peers (e.g. Volkswagen):
  - EV of all the companies identified
  - For example, EV_{VW}
  - Multiples of the firms selected based on their EV, Sales, EBITDA and EBIT
  - For example
    - $EV_{VW}/Sales_{VW}$
    - $EV_{VW}/EBITDA_{VW}$
    - $EV_{VW}/EBIT_{VW}$

- Select a range of multiples by considering the the mean, maximum and minimum of the multiples calculated

The industry comparables method applies the multiples of the peer group to a company's sales, EBITDA and EBIT to determine its EV

- Returning to the example of Tranquillity
- The multiples of other tobacco companies are computed

Company	EV/Sales	EV/EBITDA	EV/EBIT
Imperial Tabacco	7.6x (a)	16.1x	17.2x
British American Tobacco	3.9x	11.2x	12.5x
Japan Tobacco	0.7x	7.3x	11.7x
Reynolds American	2.4x	8.3x	8.9x
United States Tobacco Company	4.7x	9.7x	10.2x
Minimum	0.7x	7.3x	8.9x
Maximum	7.6x	16.1x	17.2x
Average	3.9x	10.5x	12.1x
Median	3.9x	9.7x	11.7x
Selected intervals	**2.4x - 5.0x**	**8.2x - 12.0x**	**9.0x - 13.0x**

These intervals are determined by personal choice and therefore in a very subjective manner

- The multiples intervals are selected subjectively since there is no right or wrong answer
- These intervals are then applied on the sales, EBITDA and EBIT of Tranquillity to find out what its EV should be
- Hence, the EV of Tranquillity according to the EV/Sales multiple would be between 10,032 and 20,900

(a) Calculated by $EV_{Imperial\ Tobacco}/Sales_{Impertal\ Tobacco}$

2 The M&A comparables (also called transaction comps) method, on the other hand, examines the price of previous transactions and calculates the implied transaction value of the company in question

- The use of transaction comps differs from using trading comps in that the value is based on the level at which transactions have taken place in the past, rather than on the calculated enterprise value

Date	Acquiror	Target	Cost (€ millions)	EBITDA multiple
Apr 06	Reynolds	Conwood	2,616	12.3x
Jul 04	Reynolds	B&W	2,238	4.7x
Jul 03	BAT	ETI	1,854	11.8x
Jun 03	Altadis	RTM	1,388	17.8x
May 02	Imperial	Reemtsma	3,807	12.7x
Oct 01	Gallagher	Austria Tabak	1,502	8.3x
Dec 99	Tabacalera	Seita	2,657	8.3x
Jun 99	BAT	Rothsman Int	6,099	7.0x
May 99	Japan Tabacco	Reynolds Int	5,968	14.0x
Aug 98	Imperial	Van Nelle	809	9.3x
Average				10.6x
Selected interval				**10.0x - 13.0x**

- The selected interval is then applied to Tranquillity's EBITDA
- In this way, it is possible to determine what value of Tranquillity should be

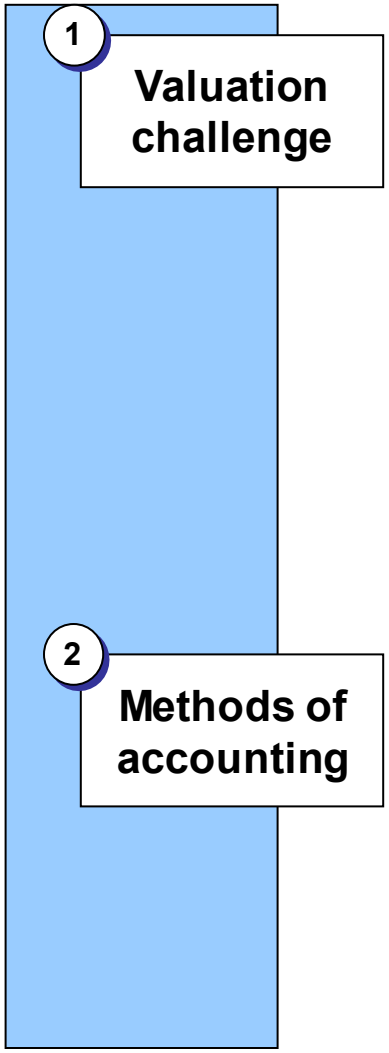
- It is needed to note that since these are actual prices paid for companies in the past, they include acquisition premia
- Therefore, to calculate the "fair prices" of Tranquillity, it is necessary to reduce the purchase prices by the acquisition premia

Topic 7: Company valuation

Sum of the parts

Sum of the parts (SoTP) is another valuation method. This could be important because most large companies operate in more than one business. Valuing a diversified company requires separate valuation for each of its businesses

- Separate cash flow forecasts with business-specific assumptions and separate discount rates that reflect the different risk of each business
- The corporate HQ needs also be valued separately and added to the sum of the business unit values
- This method is often not taught or covered in valuation textbooks perhaps because it is akin to DCF or multiples
- Nevertheless, there are 2 challenges that should be noted



Description

- The whole can be worth more or less than the sum of the parts
- The degree of ownership and control by the parent determines the accounting treatment different businesses receive in the parent's consolidated financial statements, which complicates the process of adding the parts together

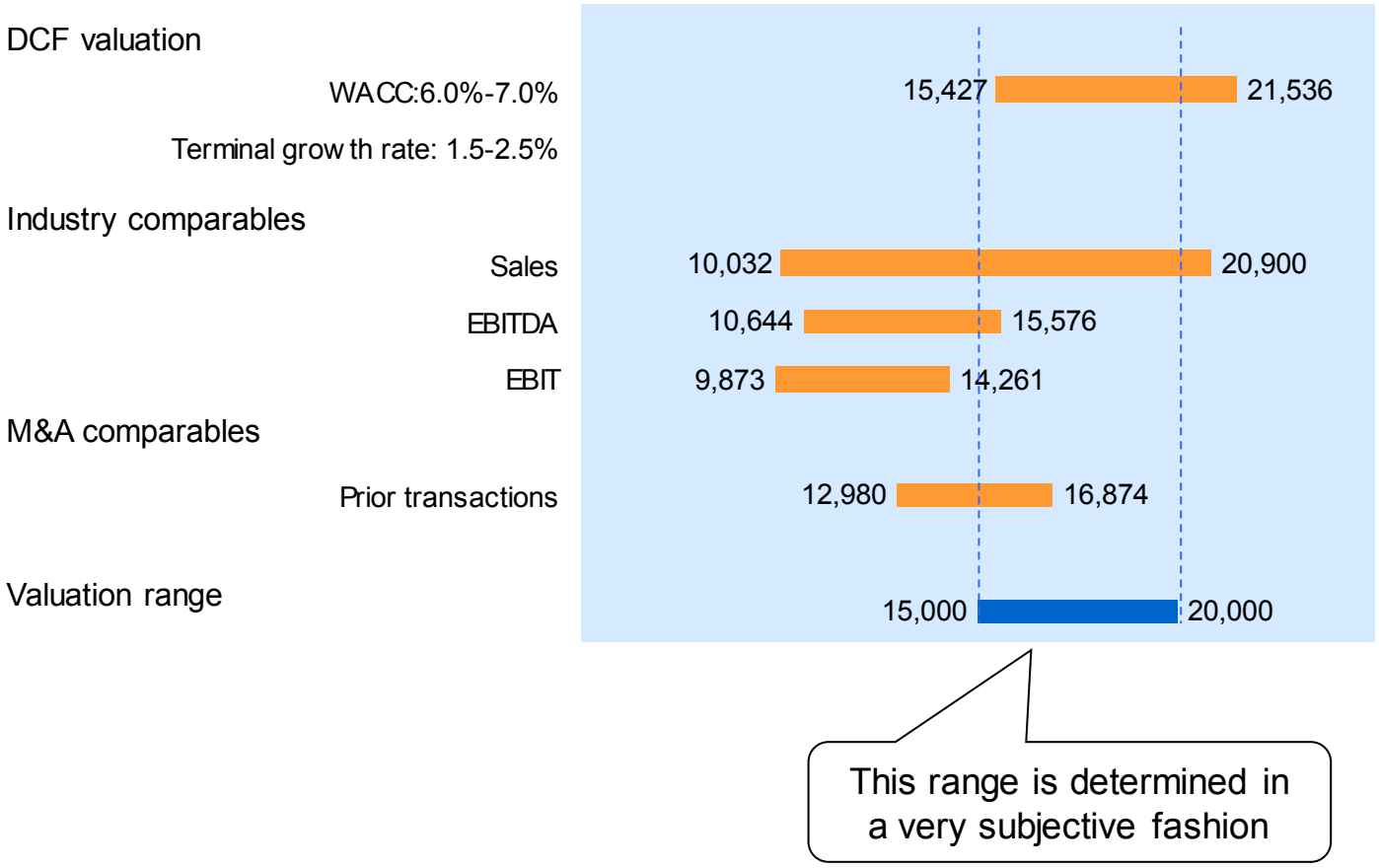
- The method of accounting differs with the amount of control a parent has over a business
- The different methods of accounting can affect the value of each business

Topic 7: Company valuation

Determination of the enterprise value

To determine the value of a firm based on all the valuation methods, we compare the values derived and draw a valuation range

Example
- Putting all the firm values based on the different valuation techniques together, we can determine the value range of Tranquillity



It is important however to bear several points in mind when doing valuation

Points	Description
1 Not an exact science	• Valuation is not an exact science. It is not the point to calculate the exact value of a firm
2 No single value	• There is no single value of a company. Instead, there is only a range of potential values
3 Inherent biases	• Each of the methods involves different biases • Consequently, the methods will lead to different results from different perspectives • Hence, in the M&A setting, it is important to fully understand the implications of various methods in order to lead the negotiations in the preferred direction
4 Parameters have impacts	• Technical parameters such as WACC and growth rate have great impacts on value and it is paramount to allocate enough time to estimate these
5 Actual value is different	• Actual value agreed upon in a transaction usually differs from the perceived value

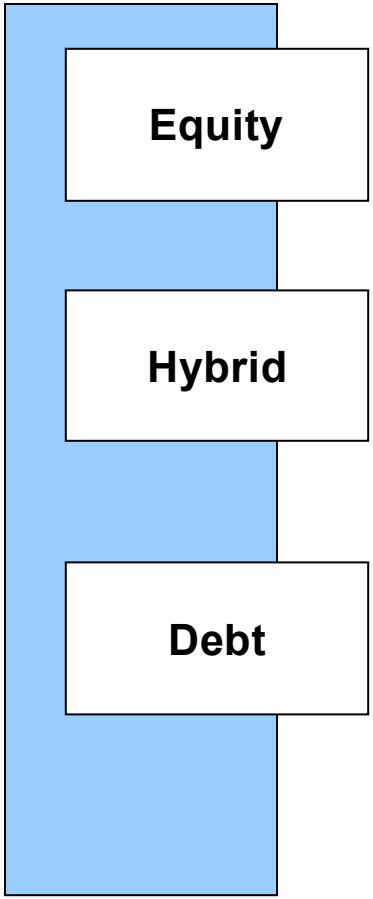
- In the end, it is the quality of the valuation that really counts
- A significant portion of the value transacted in the end – if not the transaction itself – depends on the negotiation skills of the participants

Topic 8: Capital structure

Different financing alternatives

A firm can be financed by various forms of financial instruments

- The capital structure of a firm is not just made up of debt and equity, even though it often focuses on 'debt versus equity'



- However, a challenge to financial managers is to find the combination of financing that maximises shareholders' value

Topic 8: Capital structure

Leverage

Leverage (or gearing) is related to the extent to which a firm relies on debt rather than the equity

- Financial leverage can bring both advantages and disadvantages

Pros

- Debt allows companies to increase the expected return on equity
- Debt is a cheaper form of financing compared to equity. This is because it provides a significant tax advantage as interest payments are tax deductible

Cons

- The more debt a firm has, the more difficult it is for the firm to fulfil its contractual obligations – too much debt can lead to a higher probability of insolvency and **financial distress**
- A firm that fails to make the required interest or principal payments on the debt is in **default**

The use of debt allows a company to achieve financial leverage that can amplify shareholders' returns and losses

Example

- This firm is considering replacing half of its equity of £4m with an equal amount of debt

CAPITAL STRUCTURE

	Equity-only	50/50
Shares outstanding	*400,000*	*200,000*
Price per share	*£ 20*	*£ 20*
Equity	£ 8,000,000	£ 4,000,000
Debt	*-*	*4,000,000*
Assets	£ 8,000,000	£ 8,000,000
Debt-equity ratio	-	1.0
Interest rate	*0%*	*10%*

- This change has a direct impact on the return to shareholders because of the leverage effect that debt can create

The use of debt allows a company to achieve financial leverage that can amplify shareholders' returns and losses (cont'd)

- Let us assume that there are 3 possible economic states that would lead to various levels of profitability
- Note that the different economic states are reflected by EBIT

Economic states and EBIT

	Recession	Normal	Boom
EBIT	£ 500,000	£ 1,000,000	£ 1,500,000

Impacts of economic states with different capital structure

Equity Only

EBIT	£ 500,000	£ 1,000,000	£ 1,500,000
Interest (@ 10%)	-	-	-
Net income (assuming no tax)	£ 500,000	£ 1,000,000	£ 1,500,000
Equity	£ 8,000,000	£ 8,000,000	£ 8,000,000
ROE	**6.25%**	**12.50%**	**18.75%**
Shares outstanding	400,000	400,000	400,000
EPS	**£ 1.25**	**£ 2.50**	**£ 3.75**

50% Equity/50% Debt

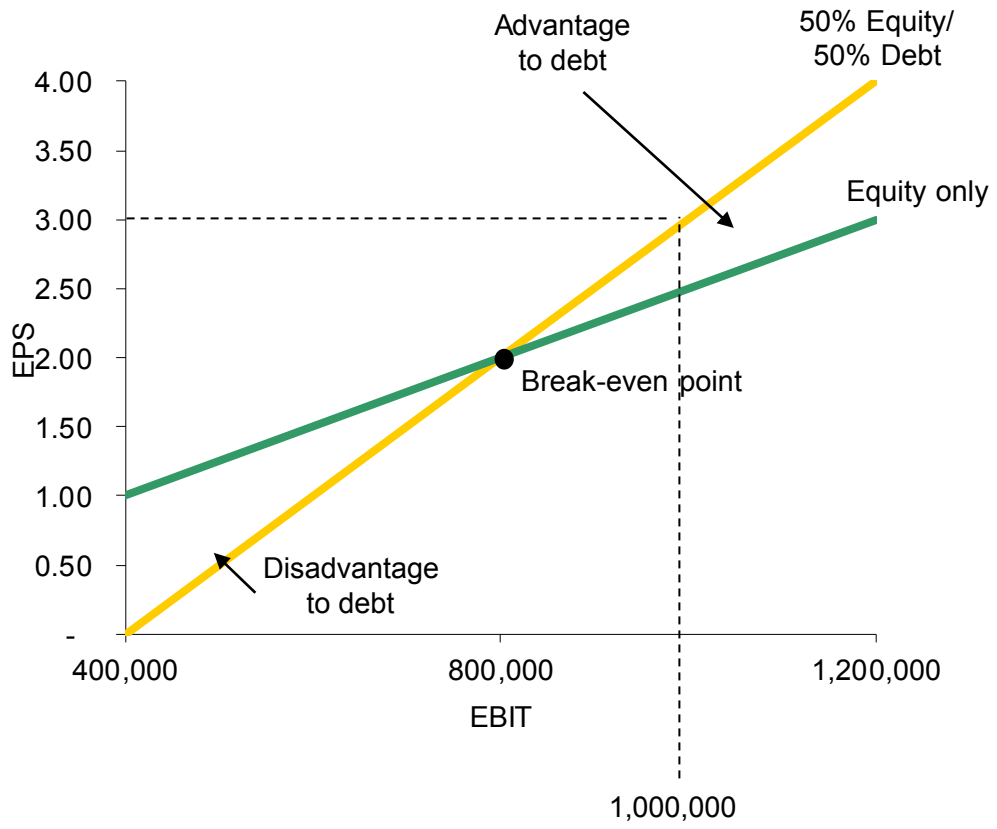
	Recession	Normal	Boom
EBIT	£ 500,000	£ 1,000,000	£ 1,500,000
Interest (@ 10%)	400,000	400,000	400,000
Net income (assuming no tax)	£ 100,000	£ 600,000	£ 1,100,000
Equity	£ 4,000,000	£ 4,000,000	£ 4,000,000
ROE	**2.50%**	**15.00%**	**27.50%**
Shares outstanding	200,000	200,000	200,000
EPS	**£ 0.50**	**£ 3.00**	**£ 5.50**

- When a firm is in debt, it can earn a higher return in good times
- But it also loses more than an all-equity firms in bad times

Shaded area denotes the highest ROE and EPS in different economic state

Graphically, it can be seen that debt will present the advantage of leverage to the firm beyond a certain break-even point

- Graphing the different economic states for the 2 capital structures



- As shown previously and in this graph, with an EBIT of £1m, the EPS will be £3.00 if the firm takes up debt but only £2.50 if it remains debt-free
- Indeed, debt will raise EPS when EBIT is greater than £800,000 (which can be called the 'Break-even point'). Thus, debt boosts the return to the shareholders
- Beneath this point, debt presents disadvantage to the firm
- This implies that a firm with debt is subject to greater volatility of earnings

Topic 8: Capital structure

MM Proposition 1

The proportions of the firm's financing from current and long-term debt and equity is called capital structure. So, here we ask the same question: Can the choice of capital structure increase the value of the firm?

- The balance sheet provides certain clues in answering this question
- From this perspective, the value of a firm is the PV of all the future cash flows that its fixed assets and operations can generate
- This should equal to the combined value of all the firm's outstanding debt and equity securities

Assets	Liabilities and Shareholders' equity
Value of cash flows from the firm's assets and operations	Market value of debt + Market value of equity
= Value of firm	= Value of firm

- If the firm changes its capital structure, say by using more debt and proportionally less equity financing, overall value should not change
- This is because it is nothing but an adjustment to the components on the right-hand side of the balance sheet
- This is the basic premise of the Modigliani and Miller ('MM') theory

According to MM, capital structure does not change the value of the firm

Example

- The firm Gondwana has total assets and operations of $1m
- It has a constant and perpetual operating income of $140,000 as well as a set of assumptions

(in 000s)

Assumptions

Perpetual and constant cash flow	*$ 140*
Total assets	*$ 1,000*
Cost of debt	*5%*
Tax rate	*0%*
Payout ratio	*100%*
Depreciation = capex	
Net working capital	*-*

Income statement at different debt level

Debt as % of capital structure	**0%**	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**
Equity	$1,000	$ 950	$ 900	$ 850	$ 800	$ 750	$ 700	$ 650
Debt	-	50	100	150	200	250	300	350
EBIT	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140
Interest	**-**	**3**	**5**	**8**	**10**	**13**	**15**	**18**
EBT	140	137	135	132	130	127	125	122
Taxes	-	-	-	-	-	-	-	-
Net income	140	137	135	132	130	127	125	122
Dividends	**140**	**137**	**135**	**132**	**130**	**127**	**125**	**122**
Total cash flows to all investors	**$ 140**	**$ 140**	**$ 140**	**$ 140**	**$ 140**	**$ 140**	**$ 140**	**$ 140**

Note that cash flows do not vary

- The total cash flows remains constant no matter how much debt Gondwana takes up – EBIT remains at $140,000
- If we increase the proportion of debt in the capital structure, the only effect resulting is a redistribution of the cash flows from shareholders to debt-holders
- Since cash flows to the firm have not changed, the total value of the firm does not change

According to MM, capital structure does not change the value of the firm (cont'd)

- The change in capital structure does not change the size of the pie (or pizza), which can be illustrated by the following:

After the ball game, the pizza man is delivering a pizza to Yogi Berra.

Pizza man: "Should I cut it into four slices as usual, Yogi?"

Yogi: "No. Cut it into eight; I'm hungry tonight."

- As Merton Miller explained when receiving the Nobel Prize for economics, "It is the size of the pizza that matters, not how many slices it is cut up into"

While capital structure does not change the value of the firm, the use of debt can improve the return to shareholders. To achieve this, shareholders do not need the firm to take on debt; they can do it all by themselves

- Even though borrowing can increase EPS, the company is **not** doing anything that the shareholders cannot do themselves
- This is because shareholders can replicate the strategy by borrowing themselves

- Following the earlier example, if the firm replaces half of its equity with debt, the EPS in different states will be:

	50% Equity/50% Debt		
	Recession	**Normal**	**Boom**
EPS	£ 0.50	£ 3.00	£ 5.50

- However, what would happen if a shareholder borrowed £20 to buy one more share and paid 10% interest?

	Recession	**Normal**	**Boom**
EPS (Equity only)	£ 1.25	£ 2.50	£ 3.75
Earnings on 2 shares	2.50	5.00	7.50
Less: interest at 10%	2.00	2.00	2.00
Net earnings on investment	£ 0.50	£ 3.00	£ 5.50

Compare this line to the impacts on the EPS at different economic states previously – they are exactly the same

- In short, investors can create financial leverage themselves to achieve different payoffs
- Therefore, it makes no difference as to whether the company borrows or not

In other words, unless there are distortions, the value of a firm must be independent of its financing policy

- As long as investors can, on their own, borrow or lend on the same terms as the firm, they are **not** going to pay more for a firm that has borrowed on their behalf
- The value of the firm will remain unchanged before and after taking on debt
- In other words, the value of the firm will be unaffected by its capital structure

- **MM's Proposition 1 (also called the MM Debt Irrelevance Proposition) therefore states that the value of a firm is the same regardless of whether it finances itself with debt or equity**
- Under ideal conditions, the firm's debt policy should not matter to shareholders
- In perfect capital markets, the value of a **levered** firm (a firm with debt) is exactly the same as the value of an **unlevered** firm (a firm with no debt, also called an all-equity firm)
- Therefore,

$$V_L = V_U$$

Topic 8: Capital structure

MM Proposition 2

Earlier, it was demonstrated that leverage can improve the earnings per share and the return on equity for shareholders. So, even though the use of debt can boost the return to the equity shareholders…

- Returning to the earlier example, we saw that replacing £4m of shares with £4m of debt does not affect the value of the firm
- Let us concentrate on the 'expected' economic state

CAPITAL STRUCTURE				
		Equity only		50% Equity/50% Deb
Shares outstanding		400,000		200,000
Price per share	£	20	£	20
Equity	£	8,000,000	£	4,000,000
Debt		-		4,000,000
NET INCOME IN THE "NORMAL" STATE IN THE EARLIER SLIDE				
EBIT	£	1,000,000	£	1,000,000
Interest (@ 10%)		-		400,000
Net income (assuming no tax)	£	1,000,000	£	600,000
RETURN TO THE SHAREHOLDERS				
Net income divided by equity		1,000,000 / 8,000,000		600,000 / 4,000,000
ROE		**12.50%**		**15.00%**
EPS	**£**	**2.50**	**£**	**3.00**

- In the 50/50 scenario, shareholders get a higher return since they obtain £600,000 from an investment of £4m (15% ROE & EPS of £3.00), whereas in the case that they are funding all the investments, they get £1m out of £8m (only 12.5% ROE and EPS of £2.50)
- Leverage has thus created a higher return for the shareholders

…leverage can also adversely affect the risk to shareholders

- However, shareholders have assumed more risk as a result of their company taking on debt
- In turn, they will demand a higher return to compensate of the additional risk undertaken

- The higher EPS and ROE for the 50/50 case therefore also reflects the increase in risk to the net income that the equity shareholders will receive
- In short, leverage increases the expected rate of return as well as increases the risk to shareholders

- Note that as these two effects offset each other, as the share price remains unchanged at £20 in both cases

The fact that the two effects offset each other is also reflected in the WACC

- Since changing the capital structure changes neither EBIT nor the firm's value, it should not affect the cost of capital either
- In short, regardless of the capital structure, the WACC will be the same

RETURN TO THE SHAREHOLDERS AND INTEREST RATE

	Equity only	50% Equity/50% Debt
ROE (r_E)	12.5%	15.0%
Interest (r_D)	0.0%	10.0%

THE WACC FOR EQUITY-ONLY

$$\frac{0}{8{,}000{,}000}\times 10\% + \frac{8{,}000{,}000}{8{,}000{,}000}\times 12.5\% = 12.5\%$$

THE WACC FOR 50% Equity/50% Debt

$$\frac{4{,}000{,}000}{8{,}000{,}000}\times 10\% + \frac{4{,}000{,}000}{8{,}000{,}000}\times 15\% = 12.5\%$$

- Similar to the argument presented above, if we increase the leverage to take advantage of the lower r_D, r_E will rise as shareholders ask for a higher return to compensate for the higher risk

Therefore, while debt will have no effect on the rate of return on equity for an all-equity firm (because there is no debt in the first place), it will affect the rate of return on equity if it is a levered firm

- Let us call the return to shareholders in an all-equity firm r_A (as in **return on assets**)
- Referring back to the discussion on the WACC above, it can be seen that the WACC does not change whatever the capital structure is (i.e. 12.5%)
- **Hence, the WACC will always be equal to r_A in a world with no tax**

$$r_A = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E$$

- Since the WACC does not change as debt increases, we can re-arrange this formula to calculate the change in r_E :

$$r_E = r_A + \frac{D}{E}(r_A - r_D)$$

- In other words, expected return on equity should be the expected return on the assets **plus** the compensation for the additional risk as a result of leverage
- This implies that $r_A - r_D$ is the 'risk spread' for the levered firm, which increases in proportion to the debt-equity ratio
- **MM2 therefore says the cost of equity rises with leverage because the risk to equity rises with leverage**

MM's Proposition 2 or MM2 states that the rate of return shareholders receive will increase as the firm's debt-equity ratio increases

- Once again, r_E is equal to r_A in a firm with no debt (also called a debt-free firm)

$$r_E = r_A = \frac{EBIT}{Market\ value\ of\ all\ securities}$$

$$= \frac{£1{,}000{,}000}{£8{,}000{,}000}$$

$$= 12.50\%$$

- So, to find out what the r_E will be if a firm loads itself with debt, we can use the formula shown before

$$r_E = r_A + (r_A - r_D)\frac{D}{E}$$

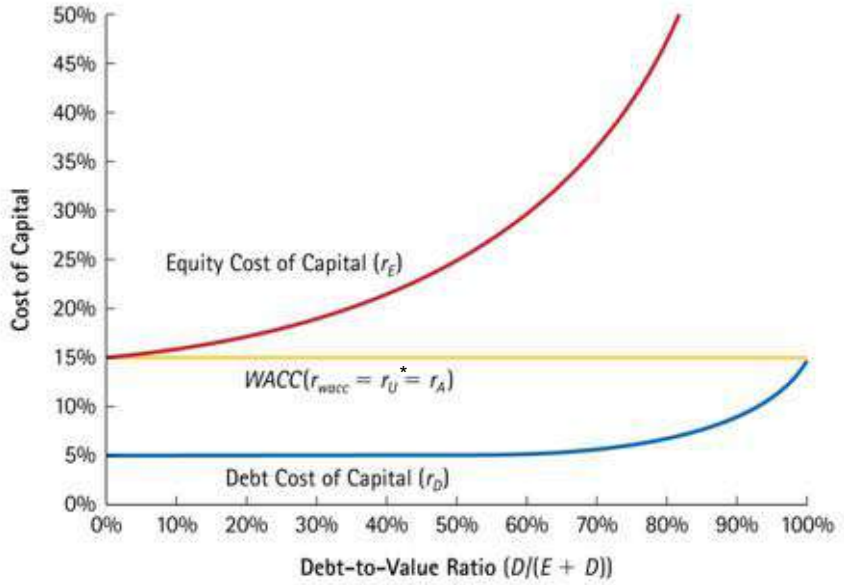
$$= 0.125 + (0.125 - 0.10)\frac{£4{,}000{,}000}{£4{,}000{,}000}$$

$$= 0.15\ or\ 15\%$$

- Note that the r_E calculated here is exactly the same as the ROE calculated earlier

MM's Proposition 2 or MM2 states that the rate of return shareholders receive will increase as the firm's debt-equity ratio increases (cont'd)

- Graphically, it can be shown that as the fraction of the firm financed with debt increases, both the equity and the debt become more risky and their respective costs of capital rise
- Yet, since more weight is put on the lower-cost debt as the amount of debt increases, the WACC will remain constant



E	D	r_E	r_D	$\frac{E}{E+D} r_E + \frac{D}{E+D} r_D$	$= r_{wacc}$
1000	0	15.0%	5.0%	$1.0 \times 15.0\% + 0.0 \times 5.0\%$	= 15%
800	200	17.5%	5.0%	$0.8 \times 17.5\% + 0.2 \times 5.0\%$	= 15%
500	500	25.0%	5.0%	$0.5 \times 25.0\% + 0.5 \times 5.0\%$	= 15%
100	900	75.0%	8.3%	$0.1 \times 75.0\% + 0.9 \times 8.3\%$	= 15%

** r_U denotes r_E with no debt. The "U" stands for unlevered*

- Notice that with the perfect capital markets, a firm's WACC is not affected by its capital structure and is equal to its r_E when there is no debt, that is,

WACC= r_A = r_E with no debt

Source: Berk and DeMarzo (2006)

Exercises

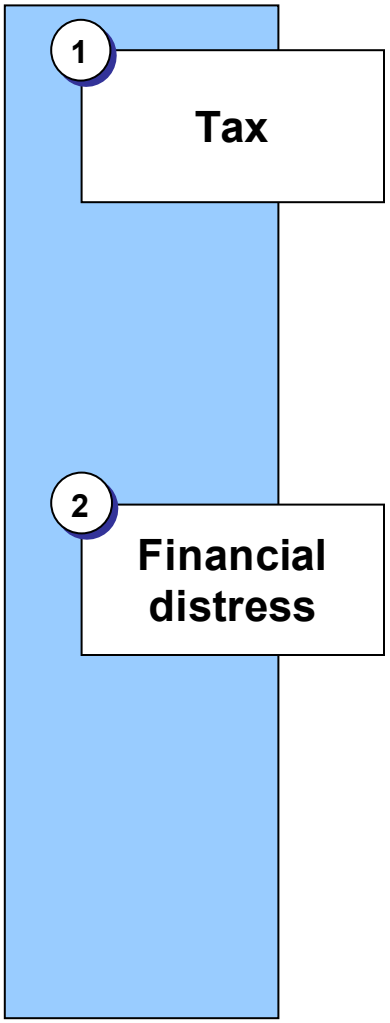
Aspects	Descriptions
Problem 1	• The Flying Corp has a market debt-equity ratio of 2. Suppose its current debt cost of capital is 8% and its equity cost of capital is 14%. Furthermore, if the company issues equity and uses the proceeds to repay its debt and reduce its debt-equity ratio to 1, it will lower its debt cost of capital to 7%. With perfect capital markets, what effect will this transaction have on Flying Corp's equity cost of capital and WACC?
Problem 2	• The Walsh Bread Company has 50 million shares that are currently trading for £4 per share and £200 million worth of debt. The debt has an interest rate of 5% and the expected return of the Walsh Bread Company share is 11%. Suppose a strike causes the price of the company to fall 25% to £3 per share. What happens to The Walsh Bread Company's equity cost of capital?

Topic 8: Capital structure

Effect of taxes

While MM's propositions are illustrative, several factors add complications

- From the discussion above, we see that while leverage increases the risk and the cost of equity, the firm's WACC, total value, and share price are unaltered by the change in leverage
- In a perfect capital market, a firm's choice of capital structure is therefore unimportant
- But in reality, firms invest significant resources in managing their capital structures
- Often, the choice of leverage is of critical importance to a firm's value and future success
- There are 2 reasons for this:



Financing	Description
Tax	• Interest on debt is usually tax deductible • This benefit is called (interest) tax shield • This is the additional amount that a firm would have paid in taxes if it did not have leverage • **Interest tax shield = interest payments x corporate tax rate**
Financial distress	• Financial distress can lead to bankruptcy and other costs including: – Bankruptcy costs (direct and indirect) – Financial distress – Debt and incentive – Cost of distress

1 Companies using debt can benefit from tax shields, thereby increasing the value of the firm

- Revisiting Gondwana in the earlier example
- With a tax rate of 40%, the total cash flow to all investors would no longer stay constant

(in 000s)

Assumptions

Perpetual and constant cash flow	*$ 140*
Total assets	*$ 1,000*
Cost of debt	*5%*
Tax rate	*40%*
Payout ratio	*100%*
Depreciation = capex	
Net working capital	*-*

Income statement at various debt level

Debt as % of capital structure	**0%**	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**
Equity	$1,000	$ 950	$ 900	$ 850	$ 800	$ 750	$ 700	$ 650
Debt	-	50	100	150	200	250	300	350
EBIT	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140	$ 140
Interest	**-**	**3**	**5**	**8**	**10**	**13**	**15**	**18**
EBT	140	137	135	132	130	127	125	122
Taxes	56	55	54	53	52	51	50	49
Net income	84	82	81	79	78	76	75	73
Dividends	**84**	**82**	**81**	**79**	**78**	**76**	**75**	**73**
Total cash flows to all investors	**$ 84**	**$ 85**	**$ 86**	**$ 87**	**$ 88**	**$ 89**	**$ 90**	**$ 91**

Note that cash flows vary according to the level of debt assumed

- The 'pizza' of the firm value must now be shared by 3 parties: shareholders, debt-holders and government
- One goal of the shareholders is to reduce the slice for the government since they are paying for it and not the debt-holders (as taxes are paid **after** interest payment)
- To do so, shareholders can choose to increase the % of debt in order to exploit the tax shield as much as possible

1 Companies using debt can benefit from tax shields, thereby increasing the value of the firm (cont'd)

- To see how a tax shield works, consider two firms that are identical in all respects, except that Firm U is unlevered and Firm L is partially financed by debt (borrowed £1,000 at 8%)
- In this case, they have the following P&L

	Firm U	Firm L
EBIT	£ 1,000	£ 1,000
Interest	-	80
EBT	1,000	920
Taxes (at 30%)	300	276
Net income	**£ 700**	**£ 644**

- Tax shield creates more value for the levered firm as value accrued to the investors is higher

	Firm U	Firm L
To shareholders	£ 700	£ 644
To debtholders	-	80
Total	**£ 700**	**£ 724**

- The extra £24 tax shield can also be calculated by £80 x 30%
- In effect, the tax shield is equal to the government paying 30% of the interest expense, with the remaining 70% or £56 paid by the shareholders

1 **The value of a firm with debt is therefore the present value of all the tax shields that the company can benefit from in the future plus the value of the firm when it is debt-free**

- If the previous example continues infinitely, the company would be earning the tax shield of £24 per year, forever
- This means that the PV of interest tax shield is equal to:

$$\frac{\text{Tax shield}}{\text{Expected return on debt}}$$

$$= \frac{\text{Tax rate x Interest payment}}{\text{Expected return on debt}}$$

$$= \frac{t \times r_D \times D}{r_D}$$

$$= t \times D$$

Expected return on debt x Amount borrowed [i.e. r_D x D]

whereas r_D is the cost of debt, D is the amount of debt and t is the tax rate

- Tax shields means the firm paying less tax, the saving of which will accrue to the shareholders
- PV (tax shield), however, depends on a firm's intention to borrow a permanent fixed amount and whether it has enough taxable income to take advantage of the tax shield

1 With tax shields taken into consideration, it is possible to develop a new version of MM's Proposition 1 that can be called MM's Proposition 1 with tax...

- MM1 states that with no tax in a perfect capital market, the value of the firm would be the same no matter what the capital structure is
- Therefore, $V_L = V_U$
- However, when tax is taken into consideration (i.e. imperfect capital markets), **MM1 with tax** proposes that the value of firm to be:

The value of all-equity firm plus the PV of all tax shields

or

$$V_L = V_U + tD$$

1 ...and MM's Proposition 2 with tax

- MM2, on the other hand, states that with no tax in a perfect capital market, the rate of return shareholders can expect increases as the firm's debt-equity ratio increases
- With tax taken into account, however, **MM2 with tax** can be written as:

$$r_E = r_A + \frac{D}{E}(r_A - r_D)(1 - t)$$

- But if tax shield is so good, why not load up a firm with debt?
- There are a number of reasons for not doing so:
  - Debt will increase the probability of distress and the associated cost
  - Profits and firm value change over time and therefore debt should not be fixed or perpetual
  - Tax rate may be at 30% but if there is a tax cut, tax shield will decrease
  - No one can be sure that the company will make perpetual profits and benefit from shield interest tax forever

1 Since shareholders have to pay personal taxes, leverage firms should assess the impacts of both personal and corporate taxes

- While benefits can be accrued to the firm as a result of taking on leverage, it is unclear whether it is beneficial for the shareholders because they have to pay personal tax
- Hence, the actual interest tax shield depends on the reduction in the total taxes – both corporate and personal – that are paid
- It is thus necessary to evaluate the combined effects of both corporate and personal taxes
- Let us say a firm has an EBIT of £1.00

	EBIT of £1.00 that is paid as…	
	interest	equity income
Corporate tax (t_c=35%)	£0	£0.35
Income after corporate tax	1.00	0.65
Personal tax (t_p=35%; t_e=10.5%)	0.35	0.65 x 0.105 = 0.07
Income after all taxes	0.65	0.65 - 0.07 = 0.58

whereas t_C is corporate tax rate, t_P is personal tax rate for interest and t_e is personal tax rate for equity

- It can be seen from this example that shareholders are taxed twice, first by corporate tax charges and second personal tax charges
- Consequently, debt-holders actually take home more than equity-holders

1 Since shareholders have to pay personal taxes, leverage firms should assess the impacts of both personal and corporate taxes (cont'd)

- Also, from the previous example it can be seen that the relative tax advantage of debt is:

0.65 – 0.58 = 0.07
(the advantage of debt financing is about £0.07 on the £)

- It is temping to conclude that the firm should put income through the branch where tax is least
- But it is not that easy because different stakeholders have different tax exemption status
- For example, pension plans do not have to worry about personal income tax
- Borrowing is not the only way to shield income against tax. Firms can
  - Accelerate write-offs for plants and equipments
  - Expense intangible assets immediately
  - Contribute to pension funds

Topic 8: Capital structure

Cost of financial distress

2 One limiting factor affecting the amount of debt a firm might use comes in the form of bankruptcy costs

- In short, the value of the firm is affected by the potential costs related to financial distress

$$V_L = V_U + PV(Interest\ tax\ shield) - PV(Financial\ distress\ costs)$$

- The firm has an incentive to increase leverage to exploit the tax benefits of debt
- But with too much debt, it has a higher risk of default and incurs financial distress costs

2 Financial distress costs cover various types of costs, including bankruptcy…

Types	Description
Bankruptcy costs	• Bankruptcies occur when shareholders exercise their right to default, allowing them to limit their liabilities and leaving all the troubles to the creditors • In this situation, the former creditors will become the new shareholders and the old shareholders are left with nothing • In contrast, shareholders of *unlimited* companies cannot simply walk away. They must pay the difference between the asset value and the bondholders' claims • Other costs including outside professionals (e.g. legal and accounting experts, consultants, appraisers, auctioneers) and administrative charges (e.g. court fees) will further reduce the value of the firm accrued to debt-holders in case of default. In other words, the current market value of the company is reduced by the PV of all fees related to outside professionals and administration • Debt-holders foresee costs they will have to pay if default occurs. Therefore, they will demand a higher payoff (in the form of higher rate of return to debt-holders), which, in turn, reduces the payoff to shareholders as well as the market value of the shares

2 ...financial distress without bankruptcy...

Types

Financial distress without bankruptcy

Description

- A company may escape a near-bankruptcy and even recover. But the mere threat of financial distress can be costly to the threatened firm.
  - Loss of customers: customers may be unwilling to purchase products, the value of which depends on future support or service from the firm. This problem is particularly acute to technology firms or airlines
  - Loss of suppliers: suppliers may be unwilling to provide a firm with inventory if they fear they will not be paid
  - Loss of employees: since firms in distress cannot offer job security with long-term employment contracts, they may have problems recruiting or even retaining existing staff
  - Loss of receivables: firms in distress tend to have difficulty collecting money that is owed to them. Knowing that the firms' resources are already spread thinly, debtors assume they may have an opportunity to avoid their obligations to the firm

2 ...financial distress without bankruptcy (cont'd)...

Types	Description
Financial distress without bankruptcy	– Fire sales of assets: companies in distress may be forced to sell assets quickly to raise cash, which means accepting a lower price than the assets are actually worth – Delayed liquidation: bankruptcy protection can be used by management to delay the liquidation of a firm that should be shut down (and continue making negative-NPV projects)

2 …and adverse behaviour of equity holders

- Conflicts of interest may occur between stakeholders when a firm is in trouble
- *If the top management hold shares in the firm*, then they may make decisions to increase the value of equity, but at the expense of the debt-holders
- This conflict is most likely to occur when the firm is in financial distress, in which case the adverse behaviour of the management team may be particularly severe
- Let us consider a company currently facing financial distress:

- The company has a loan of debt that is due at the end of this year
- Its value should be £2,000
- Clearly the company is in financial distress because the market value of the company's asset (and hence the value of the debt) is currently only worth £1,800

Current Situation

Assets	£1,800	Debt	£1,800
		Equity	-
Total assets	£1,800	Total value	£1,800

2 …and adverse behaviour of equity holders (cont'd)

Bet the debt-holders' money

- **In financial distress, the management team may choose to invest in risky projects**
- Consider that there is a project that has only a 50/50 chance of succeeding with no upfront investment
- If it turns out to be a success, the value of the firm will go up to £2,500
- But if it fails, it will fall to £300

Since the project has a 50% chance of success, the expected outcome of the investment would be:

Expected Value

Assets	£1,400	Debt	£1,150
	-	Equity	250
Total assets	£1,400	Total value	£1,400

Current Situation

Assets	£1,800	Debt	£1,800
		Equity	- (b)
Total assets	£1,800	Total value	£1,800

(a)

WIth the different outcomes…

Success

Assets	£2,500	Debt	£2,000
	-	Equity	500 (c)
Total assets	£2,500	Total value	£2,500

Failure

Assets	£ 300	Debt	£ 300 (d)
	-	Equity	-
Total assets	£ 300	Total value	£ 300

- Comparing the expected outcome of the new project with the current situation, the value of the firm's will drop by £400 (a)
- But if the managers do nothing, the shareholders will get nothing (b)
- However, if they go ahead with the project, they will potentially receive £500 after paying off the debt **if** it turns out to be successful (c)
- If not, it will be the debt-holders who bear the cost (d)
- Therefore, when a firm faces financial distress, shareholders can gain by making risky investment with debt-holders' money

2 …and adverse behaviour of equity holders (cont'd)

- There are other adverse behaviours, including:

Cash-in-and-run

- Shareholders refuse to put new money in but they are happy to take money out
- For example, a company has a piece of equipment that can sell for €250 at the beginning of the year, but it will need this equipment to continue normal operations during the year. Without it, the firm will have to shut down some operations and the firm value will drop to only €8,000 from €9,000. Although selling the equipment reduces the firm value by €1,000, this could be born by the debt-holders. However, by selling the equipment, it is the shareholders who can gain the €250 from the sales

Playing for time

- Shareholders delay the debt-holders from forcing them to sell up the assets by misleading the latter party

2 …and adverse behaviour of equity holders (cont'd)

- These adverse behaviours lead to poor decisions that are the agency cost of borrowing
- But lenders are not gullible and can anticipate such behavioural problems

- When debt-holders sense that games are being played at their expense, they can protect themselves by
  - Having veto power over potentially dangerous decisions
  - Establishing controls that limit dividends on transfer of wealth to shareholders
  - Limiting borrowing and disallowing payout on more than it earns
  - Demanding restrictive rules of assets, access to financials, and monitoring a firm's performance (this is called **debt covenants**)
- All of these can be captured in contracts. But these are agency costs that the shareholders will have to bear

- But perhaps the largest cost stems from constraints imposed on operating and investment decisions

2 Costs of distress also result from intangible assets

- The financial costs are also different between tangible and intangible assets
- Cost of bankruptcy for tangible assets is much lower than that for intangible assets because it is much easier to sell off physical assets
- The value of many organisations depend on the intangible assets
- As a result of distress,

- Key talented staff will have higher probability of defection
- Special guarantee for services may have to be provided to customers

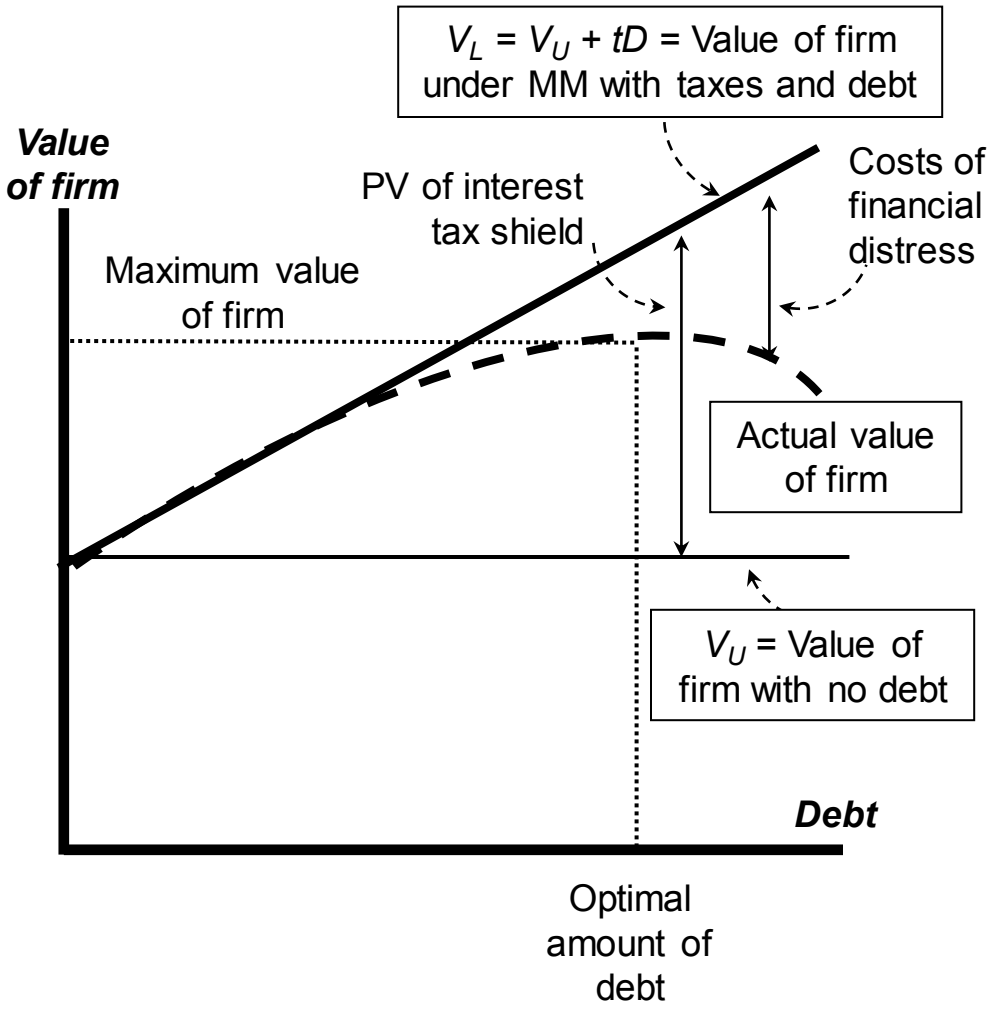
- One should not only think about the probability that borrowing will bring trouble
- One should also think about the value that may be lost if trouble comes
- This may explain why companies that depend on intangible assets to succeed have low debt ratios

Topic 8: Capital structure

Theories of capital structure

So, how do firms decide the appropriate mixture of debt and equity? Two theories have been proposed to explain the choice. The first one is called the trade-off theory of capital structure...

- MM argue that the firm's value rises with leverage in the presence of corporate taxes
- Since this relationship implies that all firms should choose the maximum debt, the theory does not predict the behaviour of firms in the real world
- So, we need other theories to explain the choice of capital structure
- The first theory is the trade-off theory because it is necessary to weight the benefit of tax shield against financial distress cost



So, how do firms decide the appropriate mixture of debt and equity? Two theories have been proposed to explain the choice. The first one is called the trade-off theory of capital structure... (cont'd)

- From the previous graph, it can be seen that

- Up to the point 'optimal amount of debt', the increase in PV (financial distress costs) from an additional £ of debt equals the increase in the PV (tax shield)
- Beyond this point, bankruptcy costs increase further than the tax shield, implying a reduction in the firm value from further leverage

- This implies that a firm's capital structure decision involves a trade-off between tax benefits of debt and the costs of financial distress
- The implication is that there is an optimal amount of debt for any individual firm
- This amount of debt becomes the firm's target debt level

So, how do firms decide the appropriate mixture of debt and equity? Two theories have been proposed to explain the choice. The first one is called the trade-off theory of capital structure... (cont'd)

Aspects	Description
What it explains	• This theory recognises that target debt ratio may vary from firm to firm – Companies with safe tangible assets and plenty of taxable income to shield out have high-target debt-equity ratio – Unprofitable companies with risky intangible assets should rely primarily on equity financing • So, contrary to MM's proposition, it **does** matter how much debt to take on
What it fails to explain	• While the trade-off theory explains many industry differences in capital structure, it does not explain as to why some of the most profitable companies tend to borrow the least

…and the second one is called pecking-order theory

- The pecking-order theory starts with the thinking that firms have access to 2 types of capital:
  - Internal capital: cash flows generated internally
  - External capital: raised outside the firm and can either be debt or new equity from new shareholders
- External capital suppliers, however, tend to know less about a company's prospects, risk and profitability than the management team
- External investors will therefore suspect that when the management team issues equity, it will only do so when the shares are over-valued
- They will also think that firms issue debt when the equity is under-valued

- Asymmetric information therefore affects the choice between internal and external financing and between issue of debt and equity. The leads to a **pecking order** in which investment is financed with:
  - First, internal funds, primarily re-invested earnings
  - Then new issue of debt
  - Finally, new issue of equity
- Managers who perceive the firm's equity is under-valued will prefer to fund investments using retained earnings or debt, rather than equity
- The converse is also true: managers who perceive the firm's equity to be over-valued will prefer to issue equity. However, due to the negative share price reaction when issuing equity, it is less likely that equity will be over-valued. As a result, managers will only issue equity as a last resort

…and the second one is called pecking-order theory (cont'd)

- Even though the pecking order theory may seem reasonable, there are several issues

- Managers do not always prefer debt to equity because asymmetric information is not always important and there are other factors at work. For instance, when a company is over-burdened with debt, it will be more reasonable to issue equity for additional financing
- High-tech, high-growth firms may also prefer equity because they have mostly intangible assets and can have high costs of bankruptcy and financial distress
- Since tax shields are not taken into consideration when managers issue debt, it is therefore (according to the theory) a bonus

…and the second one is called pecking-order theory (cont'd)

Aspects	Description
What it explains	• This theory explains why the most profitable firms generally borrow less: it is not because they have low-target debt, but because they have sufficient retained earnings and do not need outside money • Less profitable firms issue debt because they do not have internal funds sufficient for their capital investment programmes and because debt financing is first on the pecking order of external financing • The theory also explains the inverse relationship between profitability and financial leverage
What it fails to explain	• It does not explain the influence of taxes, financial distress, security issuance costs, agency costs, or the set of investment opportunities available to a firm upon that firm's actual capital structure • It ignores the problems that can arise when a firm's managers accumulate so much financial slack that they become immune to market discipline

Topic 9: Dividend policy

Dividend payment and share purchase

A firm can use its free cash in 2 ways: 1) investing and accumulating it or 2) paying it out. For the latter, there are 2 alternative payout policies. The first is to pay dividend to its shareholders

1

- Dividend refers to the cash distribution of earnings
- There are several types of dividend

- Public companies usually pay **regular cash dividends** and sometimes even an **extra cash dividend**
- **Stock dividend** refers to paying out dividend in shares (stock split). In effect, this is a stock split because if you double the number of shares, the dividend per share will be cut in half – the total amount of payout as dividend was the same just before and just after the split
- **Special dividend** is similar to regular dividend but the name usually indicates that this dividend is viewed as a truly unusual or one-time event and will not be repeated. The amount is usually larger than regular dividend
- **Liquidating dividend** usually means that some or all of the business has been liquidated or sold off (i.e. return of investment rather than return on investment)

1 Companies follow a defined procedure when paying dividends

Important dates	Definition	Example
a **Declaration date**	• The date on which the board of directors passes a resolution to pay a dividend	• On July 20, 2004, the board of directors at Microsoft passes a resolution and declares that it will pay a dividend of $3 per share
b **Ex-dividend date**	• The date 2 business days before the date of record • Establishes those individuals entitled to a dividend	• November 15, 2004 is the ex-dividend date • Before this date, the share is said to trade "with dividend" or "cum dividend" • Anyone who purchases Microsoft shares on or after this date will not receive the dividend
c **Date of record**	• Also called the record date by which a holder must be on record to be designated to receive a dividend	• Based on its records, the firm prepares a list on November 17, 2004 of all individuals believed to be shareholders. These are holders of the record, and November 17, 2004 is the date of record
d **Date of payment**	• Also called payable date • The date the dividend is paid	• Cheques are mailed out on December 2, 2004

1 Companies' decision on the choice of the size of dividend are based on various premises

Premises	Description
a **Long-run target**	• Mature firms with stable businesses generally pay out a high proportion of earnings while growth companies have low payouts (if at all)
b **Focus on the change**	• Managers focus on dividend changes and not on absolute levels. Thus, paying a £2 dividend is an important decision if last year's dividend was £1, but not the case if it was £2
c **Dividend smoothing**	• Firms adjust dividends relatively infrequent, and dividends are much less volatile than earnings • This practice of maintaining relatively constant dividends is called dividend smoothing • As a result, transitory earnings changes are unlikely to affect dividend payouts
d **No reverse in changes**	• Managers are reluctant to make dividend changes that may have to be reversed • They are particularly worried about having to rescind a dividend increase

- However, firms may choose to repurchase shares instead
- This happens when they have accumulated a large amount of unwanted cash or wish to change their capital structure by replacing equity with debt

2 Rather than paying dividends, companies can buy back their shares

2

- Share repurchase refers to the buy back of some of the outstanding shares
- There are 4 ways to do so

Methods	Description
a **Open market**	• By far the most common method, a firm announces that it plans to buy its share in the open market and then proceeds to do so like any other investors
b **Tender offer**	• A company offers to buy back a stated number of shares at a pre-specified price during short period of time – generally within 20 days • The price is typically set between 10% and 20% above the current market level to attract people to sell • Shareholders can then choose to accept or reject this offer
c **Dutch auction**	• A company lists different prices at which it is prepared to buy back shares. Shareholders submit offers declaring how many shares they wish to sell at each price and the company then pays the lowest price at which it can repurchase the desired number of shares
d **Direct negotiation**	• Also known as targeted repurchase • This refers to the direct negotiation with a major shareholder

The choice of payout policy is related to information signalling or the information a firm wants to communicate to investors. Dividend payout and share repurchase demonstrate that different signals can lead to different market reactions

Aspects	Description
Dividend payment	• Investors assess whether a firm's performance by its dividend policy • When a firm increases its dividend, it sends a positive signal to investors that management expects that the company can afford the higher dividend for the foreseeable future • If the company cannot maintain its dividend policy, i.e. cutting its dividend, shareholders will see the company as not doing well • The idea that dividend changes reflect managers' views about a firm's future earnings prospects is called the **dividend signaling hypothesis** • It is therefore no surprise to find that a higher dividend prompts a rise in the share price, whereas a dividend cut results in the fall in price • Notice that it is the change, and not the level, of dividend that matters most to shareholders • A company should also pay dividend if it cannot in invest in any project profitably

The choice of payout policy is related to information signalling or the information a firm wants to communicate to investors. Dividend payout and share repurchase demonstrate that different signals can lead to different market reactions (cont'd)

Aspects	Description
Share repurchase	• Companies buy back shares when they have accumulated more cash than they can invest profitably or when they wish to increase their debt levels (this should apply to dividends as well) • Managers are less committed to share repurchase than dividend payment, not least because it is a one-off event • Unlike dividend payout, firms do not have to smooth their repurchase activity from year to year • Share repurchase depends on the market price of the share. Managers are more likely to buy back shares when they are under-valued. But this may signal to the shareholders that the firm is under-valued • At the same time, share repurchase can also represent a sign of confidence in the firm's future for the investors *if* senior managers are committed not to sell their own shares in the share repurchase activity • Moreover, shareholders may welcome share repurchase as they are relieved to see companies paying out the excess cash rather than investing them in unprofitable projects

Topic 9: Dividend policy

Does dividend policy matter?

If a firm decides to pay cash to its shareholders, does it matter which payment policy to choose? The answer depends on whether capital markets are perfect or imperfect

- While the signals that different payout policies give can affect share price, does payout policy – the choice of paying shareholders through dividends or share repurchase – *change the value of a firm* in addition to simply signaling its value?
- 2 answers have been put forward:

1. Payout policy does not matter in perfect capital markets
2. Payout policy does matter in imperfect capital markets

1 Rather than issuing shares, what happens if cash is used to pay the extra dividend? The answer is the same – no extra value will be created for the shareholders

- Before the cash dividend, the company Laurasia has £300,000 cash
- Laurasia can choose to pay dividend by
  - (a) issuing shares or
  - (b) using its cash

BEFORE EXTRA DIVIDEND PAYOUT

Shares outstanding		100,000			
Balance sheet (market values)					
	Cash	£ 300,000	Debt	£ -	
	FA	700,000	Equity	1,000,000	
	Total	£ 1,000,000	Total	£ 1,000,000	
Price per share		**£ 10.00**			

(a) ISSUING SHARES TO PAY DIVIDEND

Dividend to be paid per share		£ *1.00*			
Financing required		£ 100,000			
Shares to be issued		11,111			
Shares outstanding after issuance		111,111			
	Cash	£ 300,000	Debt	£ -	
	FA	700,000	Old Sh. Eq.	900,000	
			New Sh. Eq.	100,000	
	Total	£ 1,000,000	Total	£ 1,000,000	
Price per share		**£ 9.00**			
plus cash dividend		£ 1.00			
		£ 10.00			

(b) USING THE CASH TO PAY DIVIDEND

Shares outstanding		100,000			
Dividend tp be paid per share		£ *1.00*			
Balance sheet (market values)					
	Cash	£ 200,000	Debt	£ -	
	FA	700,000	Equity	900,000	
	Total	£ 900,000	Total	£ 900,000	
Price per share		**£ 9.00**			
plus cash dividend		£ 1.00			
		£ 10.00			

- In both cases, the share price will become £9 since £1 is paid out as dividend
- The difference lies in the fact that in (a), there are more shareholders owning the firm and in (b), the value of the firm comes down as some of the cash is used up
- In both cases, however, the value accrued to each shareholder is £9 (share) plus £1 (dividend), which is the same as before the extra dividend payout

1 Can extra value be created by using cash to buy back shares instead? The answer is no. The value of a share does not change before or after (cont'd)

- Going back to the earlier example of Laurasia, which can use some of its £300,000 cash available to buy back shares instead of paying it out as dividend

BEFORE SHARE REPURCHASE

Shares outstanding			100,000		
Balance sheet (market values)					
	Cash	£	300,000	Debt	£ -
	FA		700,000	Equity	1,000,000
	Total	£	1,000,000	Total	£ 1,000,000
Price per share		**£**	**10.00**		

USING THE CASH TO REPURCHASE SHARES

Shares outstanding			100,000		
Shares to be repurchased			*10,000*		
Balance sheet (market values)					
	Cash	£	200,000	Debt	£ -
	FA		700,000	Equity	900,000
	Total	£	900,000	Total	£ 900,000
Price per share		**£**	**10.00**		

- While the value of the firm has shrunk, the value of each share has remained unchanged

1 Therefore, it can be seen that in perfect capital markets the firm's choice of payout policy is irrelevant and does not affect the initial share price

- **Paying extra dividend**
  - If the company increases the total amount of the payout, the extra cash must be clawed back from the shareholders by a new issue of share
  - Alternatively, it can use its cash but this is offset by a corresponding decrease in the firm's assets
- **Share repurchase**
  - If the company chooses to hold the total payout constant, any increase in the dividend payment must be offset by a corresponding reduction in the cash that shareholders receive by a repurchase of their shares

- MM's conclusion is that in a perfect capital market, firm is indifferent between a dividend payment and a share repurchase

2 However, payout policy *does* matter if capital markets are not perfect. And in reality, they are imperfect

- But payout policy is only irrelevant in *perfect* capital markets, the assumption of which ignores many forms of cost including:

- Floatation costs – selling new shares are expensive
- Taxes – different payments to the state
- Adverse signaling implications

2 Taxes are an important market imperfection that influences a firm's decision to payout policy

- Shareholders typically must pay taxes on the dividends they receive as well as capital gains taxes when they sell shares
- But do taxes affect investors' preferences for dividends versus share repurchases?

- If dividends are taxed at a higher rate than capital gains, shareholders will prefer share repurchases to dividends. The opposite is also true
- Even if there is no difference between the tax rate, the fact long-term investors can defer capital gains until they sell may mean that there is potentially tax advantage for share repurchases over dividends

2 Investors' preference between dividends and capital gains also affect a firm's choice of payout policy

Reasons	Description
1 **Clientele effects**	• The difference in tax preferences creates clientele effects, in which the dividend policy of a firm is optimised for tax preference of its investor clientele • Individuals in the highest tax brackets have a preference for shares that pay no or low dividends, whereas tax-free investors and corporations have a preference for shares with high dividends • Some financial institutions are legally restricted from holding shares that lack established dividend records such as trusts and endowment funds
2 **Steady income stream**	• Investors may look to their portfolios for steady source of cash to live on • In theory, this cash could be generated from shares paying no dividends at all because the shareholders can just sell off a small fraction of holdings from time to time • But that can be inconvenient and lead to heavy transaction cost
3 **Agency cost**	• Investors want managers to pay out rather than risking them investing in unprofitable projects and/or using the cash to build their empire • Managers may then select the payout policy of share repurchase as it does not require them to return the "surplus" cash to shareholders

Topic 10: Derivatives

Introduction of derivatives

Derivative securities (or more simply, derivatives) are securities, the prices of which are determined by, or "derive from", the prices of other securities. In other words, it is a financial instrument whose pay-offs and values are derived from, or depend on, something else

- When a firm uses derivatives to reduce its risk exposure, it is called hedging
- Derivatives can also be used to merely change or even increase the firm's risk exposure which is called speculating
- There are different types of derivatives

1. Options
2. Forwards and futures
3. Swaps

- Our focus here is options

Topic 10: Derivatives

Forwards, futures and swaps

Forward and future contracts carry the obligations to go through with the agree-upon transaction. A forward is an agreement by two parties to sell an item for cash at a later date

- The price is set at the time the agreement is signed
- Cash changes hand on the date of delivery
- Forward contracts are generally not traded on organised exchanges

February 1	Date when book arrives
You (Buyer)	**You (Buyer)**
1) Agree to pay the purchase price of £10 2) Agree to receive book when book arrives	1) Pay purchase price of £10 2) Receive book
Bookstore (Seller)	**Bookstore (Seller)**
1) Agrees to give up book when book arrives 2) Agrees to accept payment of £10 when book arrives	1) Gives up book 2) Accepts payment of £10

- It is necessary to note that cash does not change hands on February 1 but rather when the book arrives
- While you are buying a forward contract, the bookstore is selling (also called writing) a forward contract
- The act of turning the book to you is known as making the delivery

A future contract, on the other hand, is similar to a forward contract but somewhat different

- Suppose you write (i.e. sell) a contract for September pork bellies at $8.70
- This would mean you agree to turn over an agree-upon number of pounds of pork bellies for $8.70 per pound on some specified date in the month of September

- You can choose to deliver the pork bellies on any day during the delivery month, i.e. September
- Since its trading involves an exchange, there is often a clearing house acting between the buyers and sellers like you

- The benefits of futures over forwards include:
  - Liquidity because they are traded – you can easily get out by selling your contract to someone else
  - The clearing house between the buyers and the sellers act as a mechanism to prevent failure to make deliveries

Swaps are close cousins to forwards and futures contracts. Swaps are arrangements between two counterparts to exchange cash flows over time

- The two basic types of swaps are:
  - Currency swaps
  - Interest rate swaps
- The underlying mechanism is similar

- Rather than agreeing to exchange £ for US$ at an agree-upon forward price at one single date, a foreign exchange swap would call for an exchange of currencies on several future dates – the parties might exchange for $2 million for £1 million in each of the next 5 years
- Similarly, interest rate swaps call the exchange of a series of cash flows proportional to a given interest rate for a corresponding series of cash flows proportional to a floating interest rate

Example

- Consider you are running a £100m portfolio with only bonds receiving a fixed interest rate of 7% and you believe that interest rates are about to rise
- You agree to "swap" the £7m interest received so that if rates do rise, so will your interest income. In this case, you pay £7m and receive LIBOR -7% x £100m. So, with the different possible scenarios:

Different scenarios of LIBOR	**6.5%**	**7.0%**	**7.5%**
Interest income (7% x £100m)	£7.0	£7.0	£7.0
Cash flow from swap [(LIBOR – 7%) x £100m]	(£0.5)	£0.0	£0.5
Total	**£6.5**	**£7.0**	**£7.5**

- Notice that the total income on the overall position – bond plus swap agreement – is now equal to the LIBOR rate in each scenario times £100m. You have effectively turned a fixed rate bond portfolio into a synthetic floating rate portfolio

Topic 10: Derivatives

Basics of options

A financial option contract gives its owner the right but *not the obligation* to purchase or sell an asset at a fixed price at some future date. The buyers use the option only if it is advantageous to do so; otherwise the option can be thrown away

- There are 2 main types of option:

Types	Description
Call option (C)	• Call option gives the owner the right to buy an asset at a fixed price during a particular period
Put option (P)	• Put option gives the owner the right to sell an asset for a fixed price

- Because an option always involves 2 parties, for every owner of a financial option, there is an option writer
- The most common type of option contracts are on shares

There are various detailed features related to options

Details	Description
Exercising	• When an option holder takes advantage of the option by enforcing the contract, it is called exercising, i.e. the act of buying and selling underlying asset via the option contract
Strike price/ Exercise price	• The price at which the holder buys or sells the share when the option is exercised
Expiration date	• The final date on which the option can be exercised. After which the option is dead
American / European options	• American options may be exercised on any date up to and including the expiration date • European options, on the other hand, allow their holders to exercise the option *only* on the expiration date
At-the-money	• When the exercise price of an option is equal to the current price of the share, the option is said to be at-the-money • If the payoff from exercising an option immediately is positive, the option is said to be "in-the-money"; if the payoff is negative, it is said to be "out-of-the-money" • When the strike price and share price are very far apart, they are referred to as "deep-in-the-money" or "deep-out-of-the-money"

- An option contract is a contract between 2 parties
- Therefore, the buyer of the option (also called the option holder) holds the right to exercise the option and has a *long* position in the contract
- The seller, holds a *short* position in the contract; given that the long side has the option to exercise, the short-side has an *obligation* to fulfil contract

Since stock options are traded on exchanges, quotes for options are published

Near-term options on Amazon as of 8th July 2009

- By convention, all traded options expire on the Saturday following the 3rd Friday of the month
- In this case, it will be 18th July

Amazon's share last traded price

Change from the previous day's last reported sale price

Last sale price

AMZN 77.03 +1.40

Jul 08 2009 @ 15:26 ET Bid 77.02 Ask 77.03 Size 1 x 3 Vol 6548487

Calls	Last Sale	Net	Bid	Ask	Vol	Open Int	Puts	Last Sale	Net	Bid	Ask	Vol	Open Int
09 Jul 75.00 (ZQN GN-E)	7.65	1.60	7.20	7.30	221	2637	09 Jul 70.00 (ZQN SN-E)	0.36	-0.18	0.36	0.38	684	11031
09 Jul 75.00 (ZQN GO-E)	3.35	0.86	3.20	3.30	943	6883	09 Jul 75.00 (ZQN SO-E)	1.30	-0.66	1.38	1.40	2394	15545
09 Jul 80.00 (QZN GP-E)	0.94	0.24	0.93	0.96	2456	9677	09 Jul 80.00 (QZN SP-E)	4.15	-1.05	4.00	4.10	700	10718
09 Jul 85.00 (QZN GQ-E)	0.22	0.07	0.19	0.21	497	26679	09 Jul 85.00 (QZN SQ-E)	8.25	-1.25	8.25	8.35	112	7215
09 Aug 70.00 (ZQN HN-E)	9.75	1.04	9.60	9.70	51	326	09 Aug 70.00 (ZQN TN-E)	2.77	-0.39	2.75	2.79	225	1979
09 Aug 75.00 (ZQN HO-E)	6.50	0.70	6.40	6.50	65	1108	09 Aug 75.00 (ZQN TO-E)	4.60	-0.55	4.55	4.60	2322	6832
09 Aug 80.00 (QZN HP-E)	4.00	0.50	3.90	4.00	172	2462	09 Aug 80.00 (QZN TP-E)	6.95	-0.95	7.05	7.15	145	2335
09 Aug 85.00 (QZN HQ-E)	2.15	0.15	2.22	2.26	833	5399	09 Aug 85.00 (QZN TQ-E)	10.15	-1.00	10.30	10.40	43	4599

Year and month of expiration

Strike/ exercise price

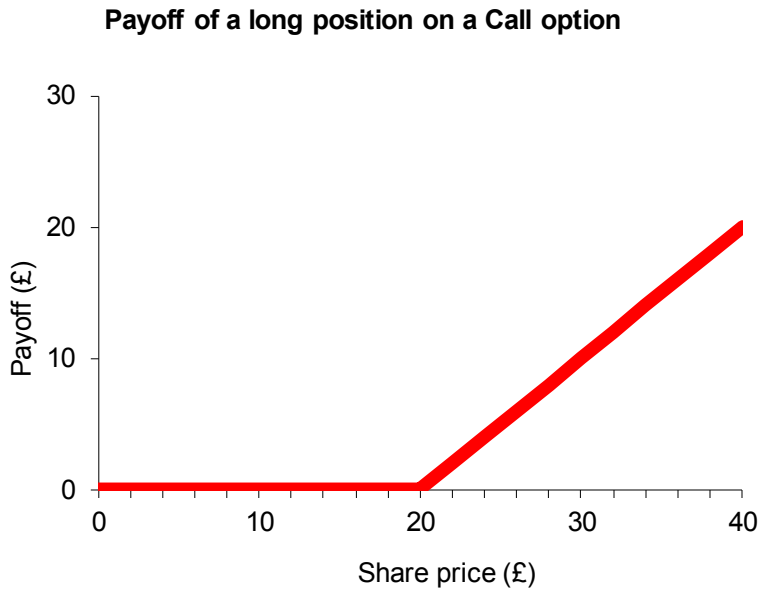
Open interest: Total number of outstanding contracts of that option

Source: Berk and DeMarzo (2011)

Let us examine options by first looking at long position on (i.e. buying) a call option …

- If the option you hold has a strike price of £20 and on the expiration date the share price is £30 (i.e. S>X), then …
  - … you can make money by exercising the call option to buy the share at £20 and then immediately sell it back to the market at £30
  - The £10 is the payoff from the option
- If the option you hold has a strike price of £20, and the share price is lower than the strike price (i.e. S<X), then…
  - … you will *not* exercise it and simply abandon it
  - In this case, the option is worthless

- We can graph the payoff of such an option



- If share price is below £20, the call option has no value and therefore no payoff; payoff only materialises when share price is greater than strike price
- So, the value of this (or any) call option must be:
  - C=max(S-X,0)

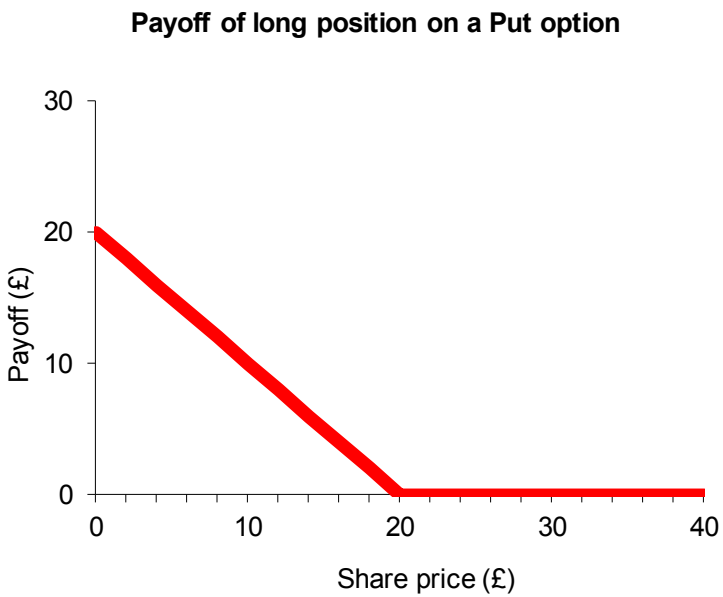
    Where C is the value of the call option, max is the maximum of the 2 qualities, S is the share price and X is the strike price
- Note that there is no maximum payoff to you

… and then at long position on (i.e. buying) a put option

- If the option you hold has a strike price of £20 and on the expiration date the share price is £30 (i.e. S>X), then …
  - … you will not exercise the option
  - In this case, this put option is worthless

- If the option you hold has a strike price of £20, and the share price is lower than the strike price (i.e. S<X), then…
  - … you will exercise the option
  - Since you will receive the strike price when the share price is trading at a price lower than the strike price, you gain the difference

- Therefore, if we graph payoff, it will be:



- So, the value of a put option must be:
  - P=max(X-S,0)

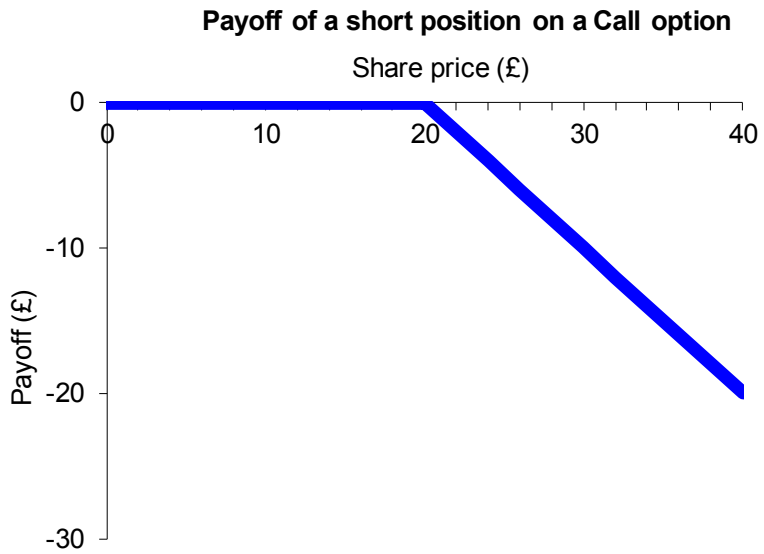
    Where P is the value of the put option
- Note that the maximum payoff you can get on a put option is X
- If the share price is equal or greater than strike price, you do not gain anything
- Payoff to you only materialises if strike price is greater than share price

Let us also look at options from the seller's perspective. First, let us examine a short position on (i.e. selling) a call option …

- If you are holding a short position in an option, you *must* fill the contract as the buyer of the contract has the option

- Just as the buyer can only receive or not lose money at expiration, you can only pay money or not lost money
- If you sell a call option, your opposite party will only exercise the option if share price is greater than the strike price (i.e. S>X)
- Therefore, you are losing money as long as share price is greater than strike price
- In other words, you are losing money if the share price is greater than the strike price of £20

- So, to graph the payoff:



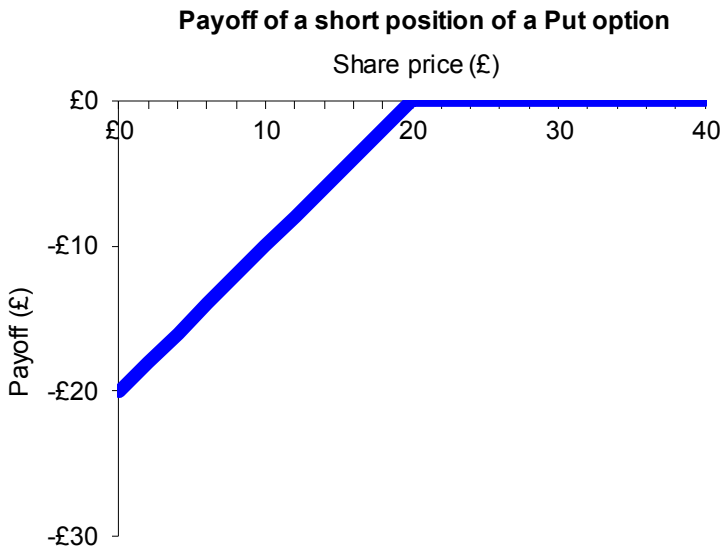
- The loss of the a short position on a call option must be:
  - C=-max(S-X,0)
- Notice that you do not lose anything until share price is greater than the strike price
- Additionally, if share price is greater than strike price, there is no maximum loss

… and then we can examine a short position on (i.e. selling) a put option

- Holders of a put option will only exercise it when the strike price is greater than share price (i.e. X>S)

- So, if you are selling a put option, you will lose money as long as this is the case
- In our example, you will lose money if share price is lower than the strike price of £20

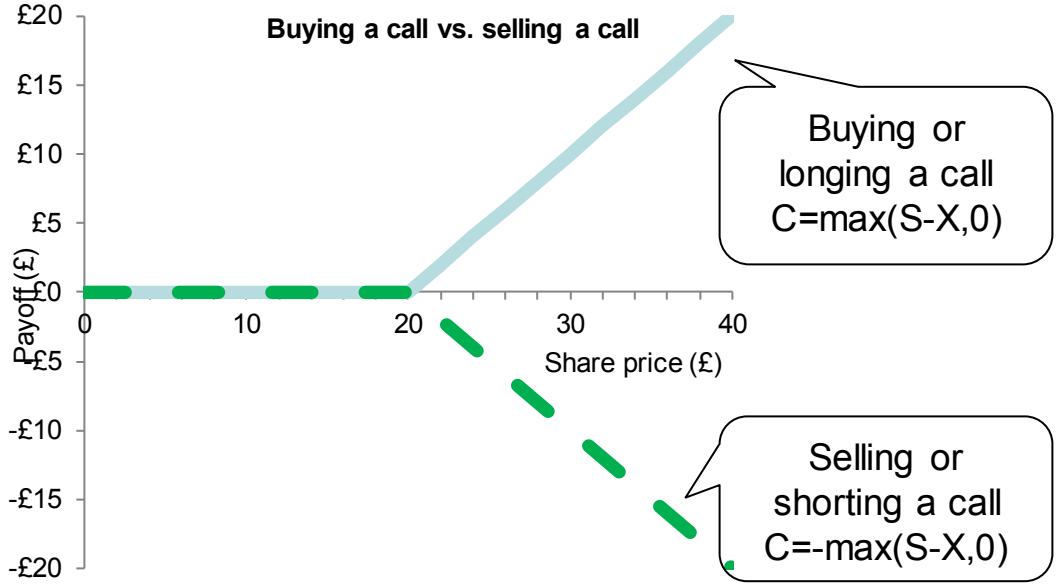
- The payoff can be graphed as follows :

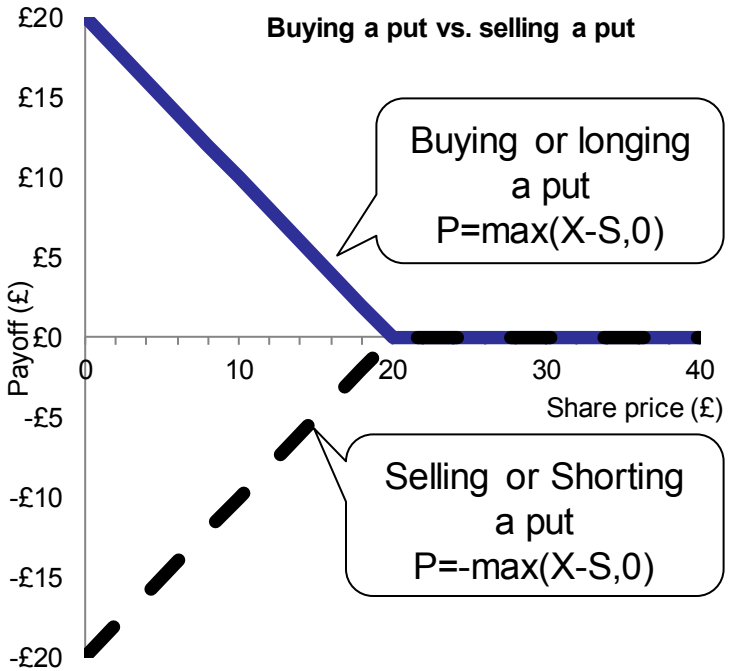


- So, the loss will be:
  - P=-max(X-S,0)
- Note that as long as share price is greater or equal to strike price, the loss is zero
- Also, unlike shorting a call, there is a limited downside loss
- In other words, there is a maximum loss of £20

This can all be extremely confusing (and it is!). One way to see how they work is to understand that they are merely the mirror image of each other

- Bringing the different positions together
- If the strike price is £20, then...





- Notice that longing and shorting options are the exact opposite
- Call options can have no maximum payoff to the buyers and no maximum loss to the sellers
- In contrast, put option has limited payoff to the buyer and limited loss to the seller

This can be all be extremely confusing (and it is!). One way to see how they work is to understand that they are merely the mirror image of each other (cont'd)

- We can compare the different payoffs

a) Gain the difference between share price and strike price with no maximum gain
or
b) no gain at all

a) Lose the difference between share price and strike price with no maximum loss
or
b) no loss at all

	Buy/Long	Sell/Short
Call	Max(S-X,0)	-Max(S-X,0)
Put	Max(X-S,0)	-Max(X-S,0)

a) Gain the difference between strike price and share price with a maximum gain equals to the strike price
or
b) no gain at all

a) Lose the difference between strike price and share price with a maximum loss of the strike price
or
b) no loss at all

- Notice that the "minus" sign in short position provides important insights
- Since the negative sign makes sense as what the buyer's gain is the seller's loss,
  - The payoff on long positions is never negative
  - On the other hand, the payoff on short positions is never positive
- So, the question now is who would want to sell options when you can only lose money? The answer lies in the fact that we need to consider the cost of options

So far, the cost of an option has not been taken into account. What would happen if this is to be included?

- As shown above, the payoffs to the sellers are never positive
- For the sellers to make a potential gain, they will have to sell the options for a fee
- For example, if the strike price of a call is $100 and the call option itself costs $14, then the payoff with profit would be different from the payoff without profit

- Note that the break-even point for the seller is $114
- The profit to the seller in this case can be up to a maximum of $14, which represents the premium that the buyer has to pay for the option

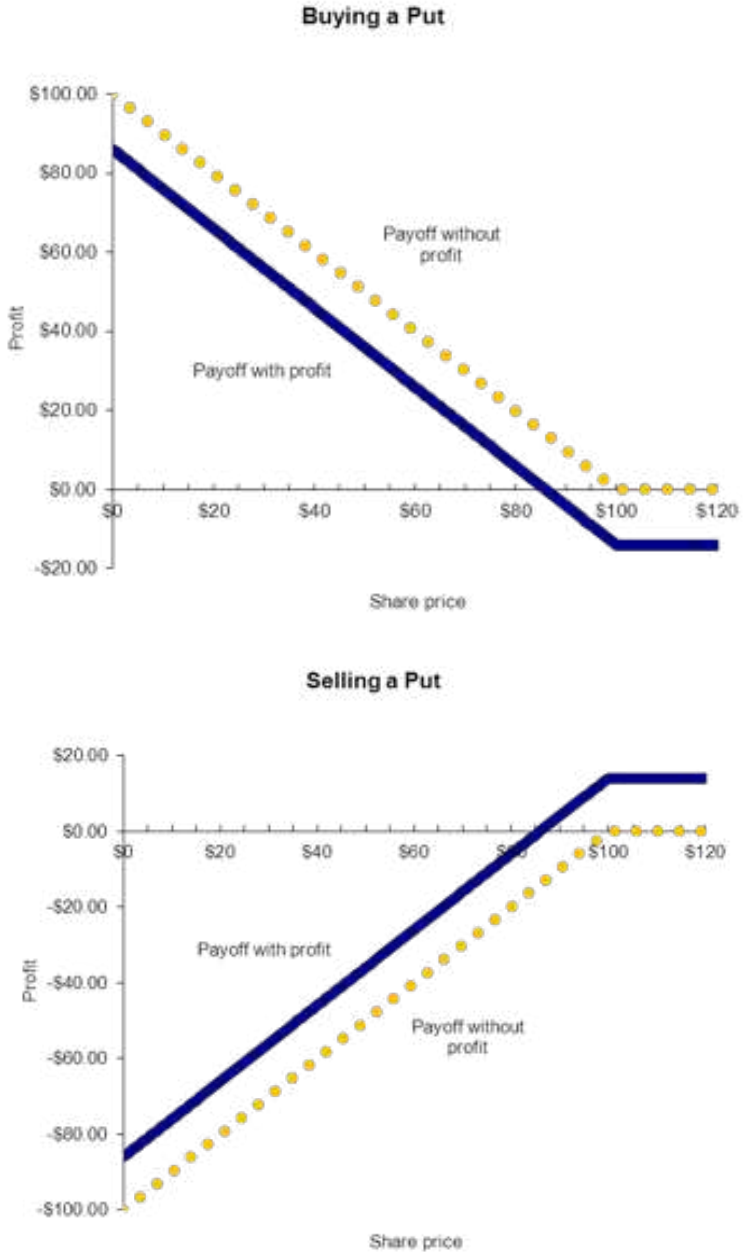
So far, the cost of an option has not been taken into account. What would happen if this is to be included? (cont'd)

- We can apply the same idea to writing a call…

- The seller will not lose all the gains from selling the put until share price hits $114

So far, the cost of an option has not been taken into account. What would happen if this is to be included? (cont'd)

- … and buying and selling a put



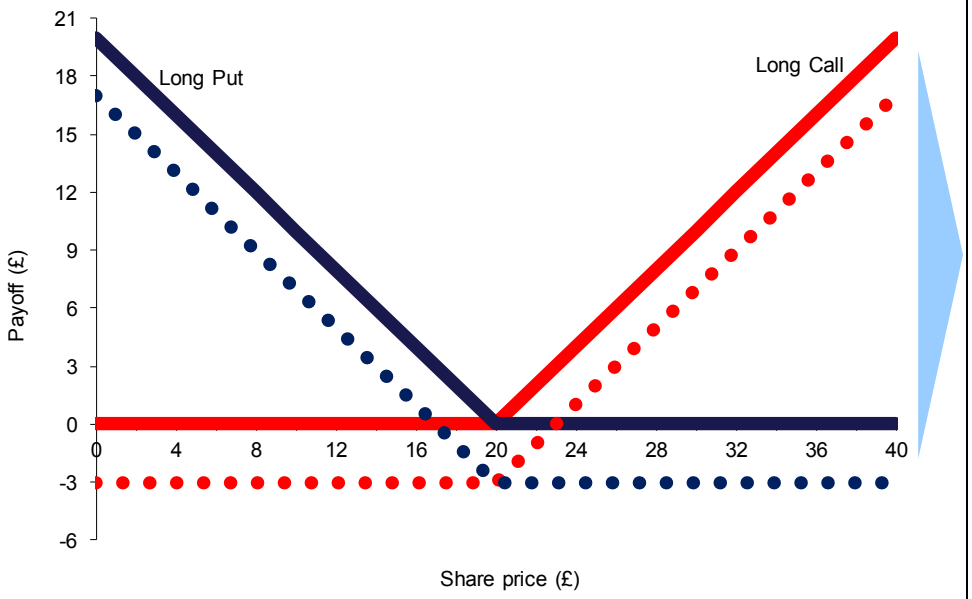
- Writing out-of-the-money puts was once considered an alternative way to generate income, as it was believed that as long as the market did not fall sharply before the option expiration, the option premium could be collected without the put buyer ever exercising the option against the seller

Topic 10: Derivatives

Combination of put and call options

Many investors combine call and put options in their portfolios to form different investment strategies. One of them is called straddle

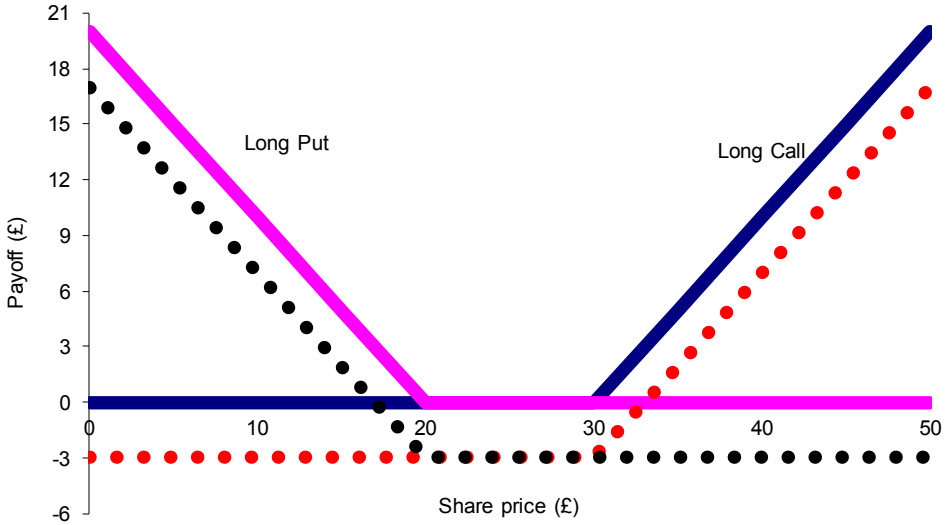
- Straddle involves buying both a call and a put option, each with the same strike price and the same time expiration
- So, the call strike price is the same as the put strike price, which, for instance, is equal to £20



- Straddle allows you to make money as long as the options do not expire at-the-money, that is, £20
- So, you are betting on the share moving a lot in price and are uncertain about the direction of the move
- The further away from strike price (i.e. the lower or the higher the strike price), the better is the payoff
- Note that since we have to pay £3 for call option, profit can only be made if S is a bit "away" from X (the dotted lines)
- Hence, the maximum you can lose is the cost of option
- This strategy is sometimes used for a share with high volatility but without necessary a view as to whether the share price will go up or down

Another common investment strategy is called strangle

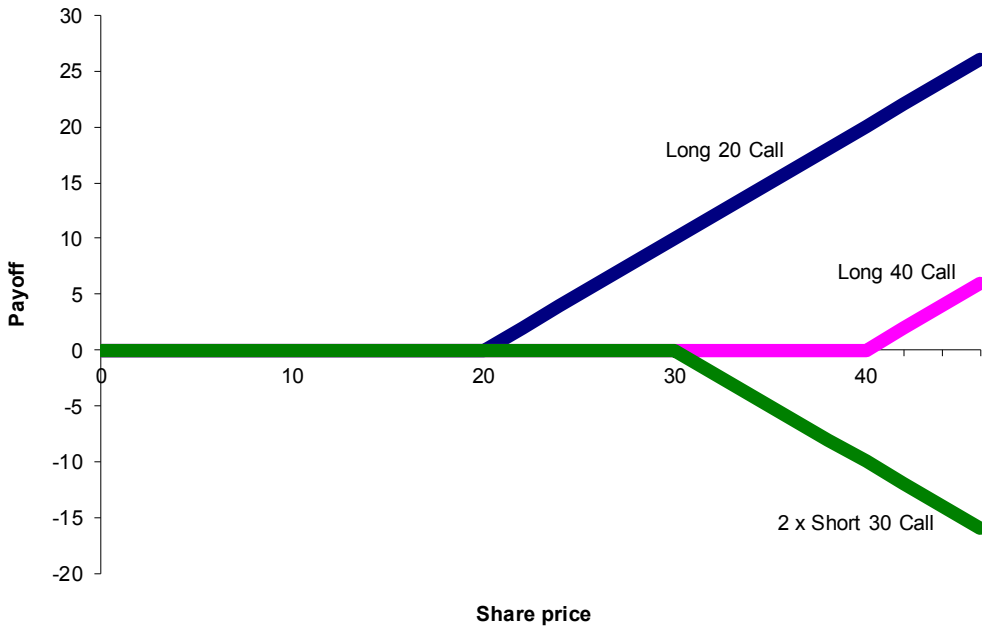
- Strangle differs from straddle by longing a call option and a put option with the call having a higher strike price (solid lines)
- Let us assume the call option with a strike price of £30 and a put with a strike price of £20 with the same expiration date



- In this case, there is no payoff if the share price is between two strike prices
- This strategy enables investor to make money when share and strike prices are far apart
- Note that the investors would lose the cost of the option of £3 if the share price is between the two strike prices (dotted lines)

Another investment strategy is butterfly spread

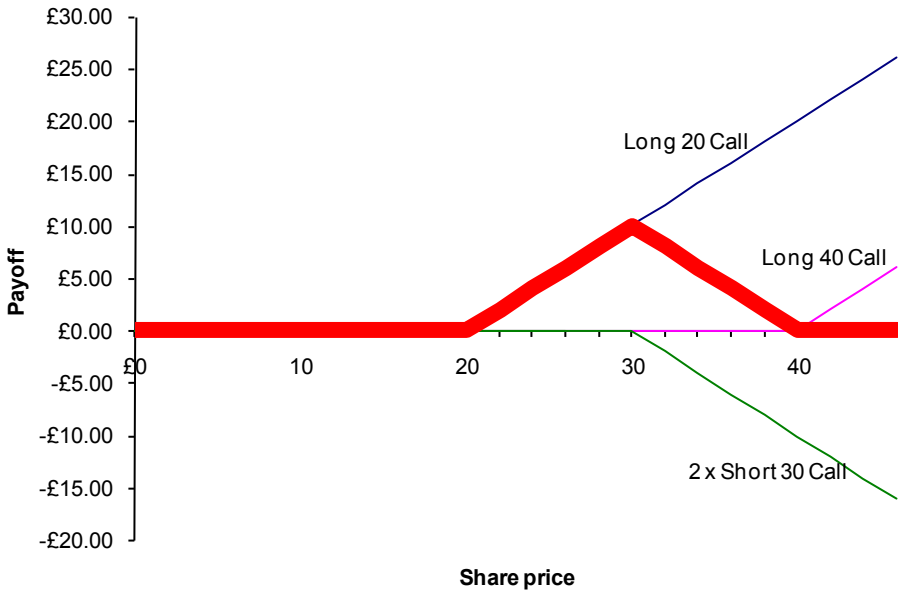
- Butterfly spread can be seen as an investment with exposure opposite to strangle – one that pays off when share price is close the strike price
- This involves longing 2 call options (1 with X=£20 and 1 with X=£40) and shorting 2 call options (both with X=£30), all with the same expiration date



- The different lines represent the payoff for each of these four options

Another investment strategy is butterfly spread (cont'd)

- The payoff for the entire combination is represented by the thick red line



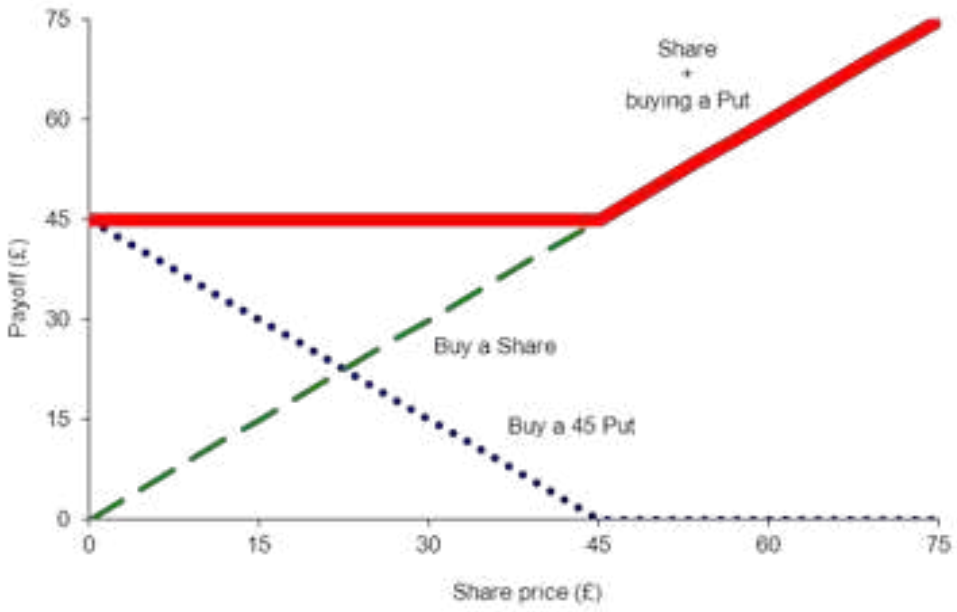
- When share price is lower than £20, all options are out-of-the-money and the payoff is 0
- When share price is above £40, the losses from the short position in the £30 calls are offset exactly by the gains from the long position of the £20 and £40 calls – the payoff is therefore 0
- This combination makes money when share price is between £20 and £40 and reaches a maximum at £30

Combination of options can also be used to insure a share against a certain level of losses. One possibility is called protective put …

- Imagine you would like to invest in a share but are unwilling to bear the potential losses beyond certain level
- A possibility to do so is to consider investing in a share and an option
- So, suppose you invest in a share and buy a put option with a strike price of £45
- The outcomes would be as follows:

	S≤X	S>X
Share	S	S
+ Put	X-S	0
Total	**X**	**S**

	S≤£45	S>£45
Share	S	S
+ Put	£45-S	0
Total	**£45**	**S**



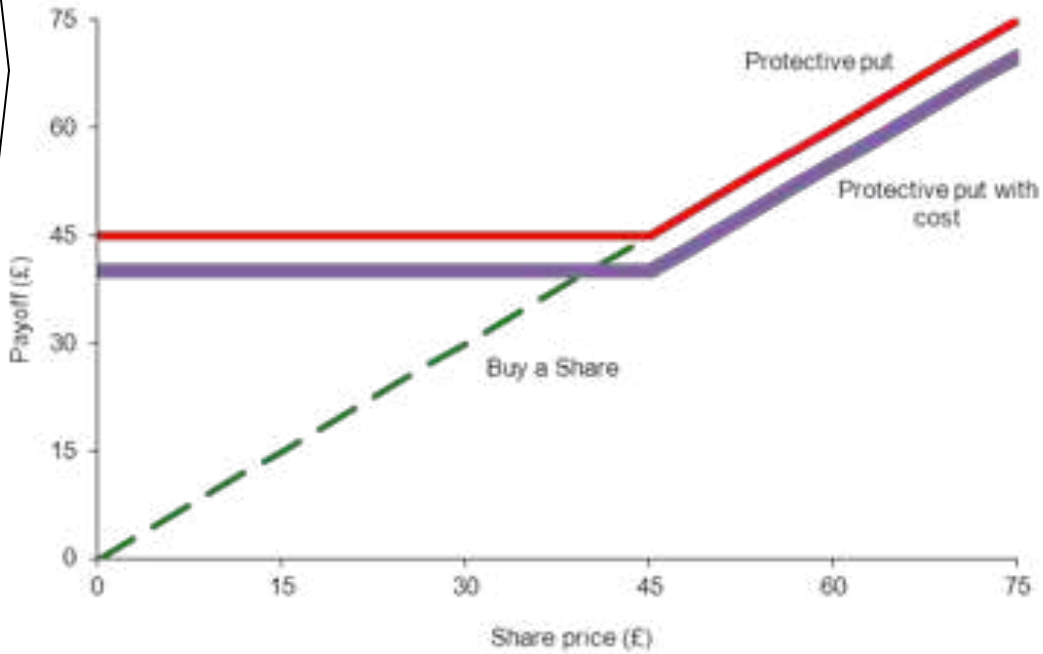
- So, whatever happens to the share price, you are guaranteed a payoff equal to the put option's strike price because the put gives you the right to sell the share for that price
- This strategy of buying a share and buying a put is called a protective put
- With this combination, if the share price is above £45 at the expiration date, you keep the share
- If the share price is below £45, you exercise the put and sell it for £45

Combination of options can also be used to insure a share against a certain level of losses. One possibility is called protective put … (cont'd)

- What happens if the cost of the put option in the above case is £5?

	S≤X	S>X
Share	S	S
+ Put	X-S	0
- Option cost	k	k
Total	**X-k**	**S-k**

	S≤X	S>X
Share	S	S
+ Put	£45-S	0
- Option cost	£5	£5
Total	**£40**	**S-£5**



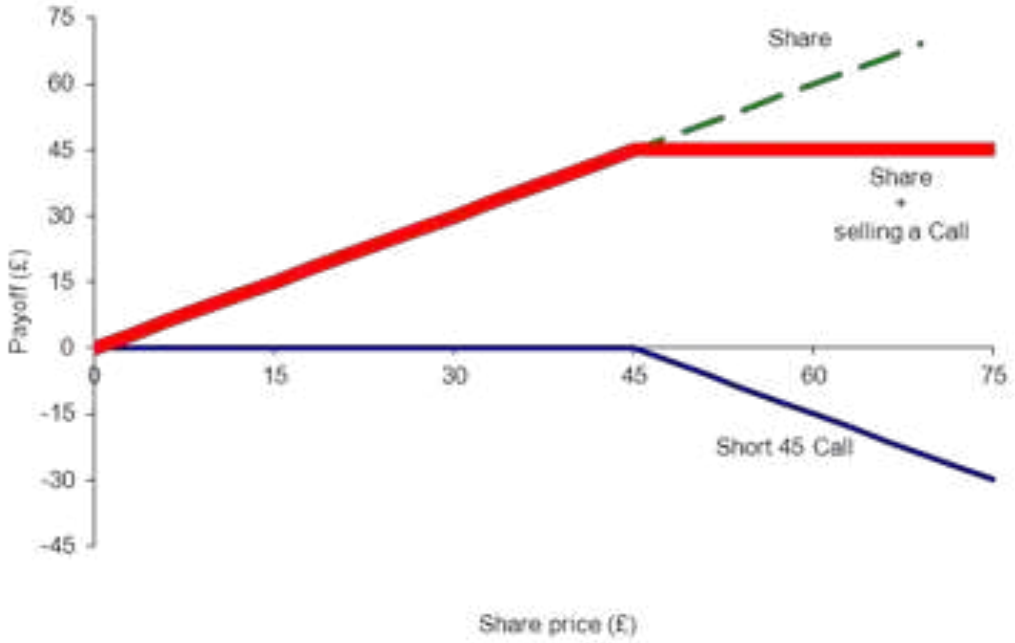
- When the share price is equal to or below the strike price, the profit will always be £40
- When the share price is above strike price, the profit is always £5 less
- In this case, the cost of the protection is that, when share price increases, your profit is reduced by the cost of the put (i.e. £5)
- From this example, it can be seen that options can be used to protect a portfolio of investments

An alternative way is called covered call

- Another way to insure against losses is to take a covered call position
- It involves 1) buying a share and 2) selling a call with a strike price of, for instance, £45

	S≤X	S>X
Share	S	S
+ Call	-0	-(S-X)
Total	**S**	**X**

	S≤£45	S>£45
Share	S	S
+ Call	-0	-(S-£45)
Total	**S**	**£45**



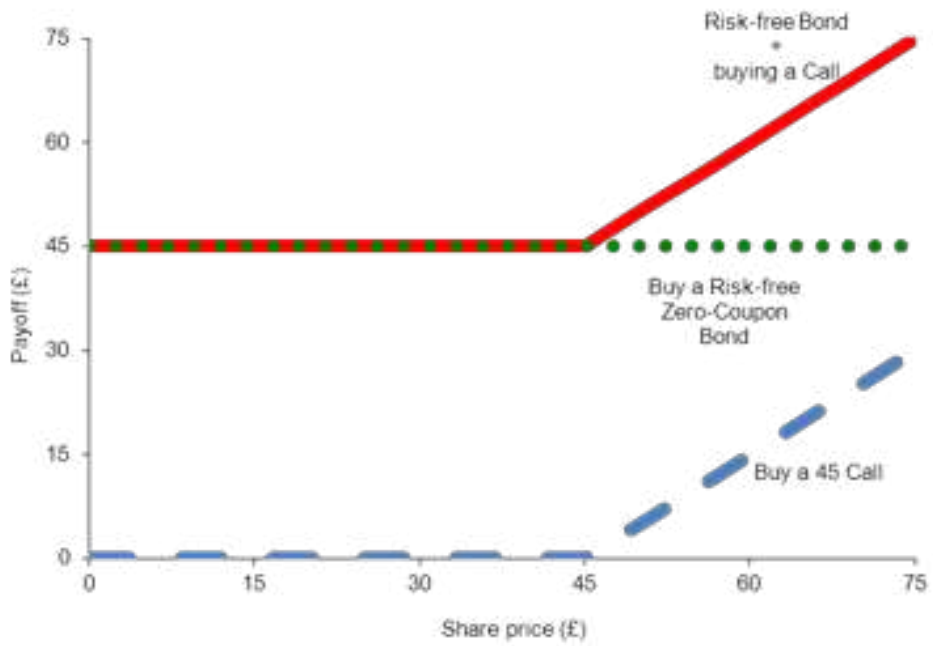
- The call is "covered" because the potential obligation to deliver the share is covered by the share held
- Note that the call value is *subtracted* because the covered call position involves writing a call to another investor who may exercise it at your expense
- This is a popular strategy for many fund managers because their portfolios are mostly made up of shares
- By taking a covered call position, they can boost their income and the gain can be used to cushion somewhat against losses on the share

It was shown earlier that a protective put involves buying a share and a put. It is possible to replicate the same effect through another combination of instruments

- Instead of holding the share plus buying a put, you can
  1) buy a call with strike price of £45 (the dotted line) and,
  2) buy a risk-free zero coupon bond with a face value of £45 that matures on the same day of the option expires (the broken line)

	S≤X	S>X
Bond	X	X
+ Call	0	S-X
Total	**X**	**S**

	S≤£45	S>£45
Bond	£45	£45
+ Call	0	S-£45
Total	**£45**	**S**



- If the share price is equal or below £45, you get the payoff from the bond
- If the price of the share is above £45, you use the proceed from the bond to buy the share for the strike price of £45 after exercising the call
- Both the tables and the graph here show that this combination of buying a call and buying a risk-free bond can achieve the same effect as the protective put strategy

Topic 10: Derivatives

Put-call parity and factors affecting option prices

The fact that holding a share + buying a put and holding a risk-free bond + buying a call achieve the same effect allows us to develop an important insight called *put-call parity*

- If 1) holding a share and buying a put and 2) holding a bond and buying a call lead to the same payoffs, then the two strategies *must have the same price and cost the same to establish*
- Presenting this concept more formally,

$$S + P = PV(X) + C$$

where:
S = share price
P = price of the put
X = strike price
PV(X) = present value of a risk-free zero coupon bond with the price of X
C = price of the call

By re-arranging it,

$$S + P = \frac{X}{(1+r_f)^t} + C$$

Where:

r_f = the risk free rate

t = the time to maturity of an option

- The relationship between the share, the bond, the call and put options is called put-call parity because it represents the proper relationship between put and call prices
- Note that it is a *very precise* relationship. It holds *only* if the put and the call have both the same exercise price and the same expiration date. In addition, the maturity date of the zero coupon bond *must be the same* as the expiration date of the options
- The importance here is that if the parity relation is ever violated, an arbitrage opportunity arises

The put-call parity, if violated, presents an arbitrage opportunity

Example
- Suppose you collect these data for a certain share

Share price	$110
Call price (1-yr expiration, X=105)	$17
Put price (1-yr expiration, X=105)	$5
Risk-free interest rate	5% per year

$$S + P = \frac{X}{(1+r_f)^t} + C$$

$$110 + 5 = \frac{105}{(1 + 5\%)} + 17$$

$$115 \neq 117$$

- This result, a violation of the parity – 115 does not equal to 117 – indicates a mispricing
- To explore the mispricing and profit from it, you can buy the relatively cheap portfolio (the S+P) and sell the relatively expensive portfolio (C+PV(X))
- In other words, if you buy the share, buy the put, sell the call and borrow $100 for 1 year (because borrowing is the opposite of buying a bond), you should be able to earn arbitrage profits